ally, feedback systems with these gain and phase margins typically yield performance close to that computed for the nominal system.

In summary, the stability margins provide information on stability robustness when the plant is subject to gain and phase perturbations. These margins are simply computed from the nominal Nyquist plot. The stability margins can also be computed from other representations of the system frequency response, for example, Bode plots and Nichol's plots. Additionally, the gain margins can be found by solving for the roots of the characteristic equation as a function of gain, or by using the Routh stability test.\(^5\)

### 5.4 Unstructured Uncertainty

The gain and phase perturbations presented above are special cases of uncertainty in the mathematical model of the plant. Many other types of perturbations exist and could be used when evaluating system robustness. In this section, uncertainty is modeled as a perturbation to the nominal plant. This perturbation is a bounded transfer function, where bounded is defined in terms of the system $\infty$-norm. This type of plant uncertainty is termed unstructured since no detailed model of the perturbation (the unknown transfer function) is employed.

#### 5.4.1 Unstructured Uncertainty Models

An unstructured perturbation can be connected to the plant in a number of ways, each generating a unique set of possible plant models. Five basic connections of the perturbation to the nominal plant model are presented: additive perturbation, input-multiplicative perturbation, output-multiplicative perturbation, input feedback perturbation, and output feedback perturbation. An additive unstructured uncertainty models the actual plant as equal to the nominal plant plus a perturbation:

$$G(s) = G_0(s) + \Delta_a(s)$$

where $\Delta_a(s)$ denotes the additive perturbation. An input-multiplicative uncertainty models the actual plant as the nominal plant plus a series combination of the perturbation and the nominal plant (the perturbation appears on the input to the nominal plant):

$$G(s) = G_0(s)[I + \Delta_i(s)],$$

where $\Delta_i(s)$ denotes the input-multiplicative perturbation. An output-multiplicative uncertainty models the actual plant as the nominal plant plus a series combination of the nominal plant and the perturbation (the perturbation appears on the output to the nominal plant):

$$G(s) = [I + \Delta_o(s)]G_0(s),$$

where $\Delta_o(s)$ denotes the output-multiplicative perturbation. An input feedback uncertainty models the actual plant as the nominal plant in series with the perturbation in a feedback loop (the feedback loop appears on the input to the nominal plant):

$$G(s) = G_0(s)[I \Delta_f(s)]^{-1},$$

\(^5\)The Routh stability test is described in most introductory control texts; for example, see Dorf [4], page 183.