The resulting perturbation has the property that
\[ \left\| \Delta_1 \right\|_\infty \leq 1; \left\| \Delta_2 \right\|_\infty \leq 1; \left\| \Delta \right\|_\infty \leq 1, \]
\( \Delta_1(s) \) is a complex perturbation, and \( \Delta_2(s) \) is a real perturbation.

A more general description of uncertainty can be obtained by allowing blocks to be purely real or adding the constraint that two or more blocks are proportional. The analysis of robustness for these cases is not treated in order to keep the notation and derivations simple. Additionally, the standard structured uncertainty model is applicable to a wide variety of systems even without these cases. For robustness analysis of systems with real perturbations, see [5, 6] and for repeated or proportional perturbations, see [7]. The remainder of this section will discuss how to analyze the stability robustness of systems with structured complex perturbations in standard form and no repeated blocks.

### 5.5.2 The Structured Singular Value and Stability Robustness

The stability of a system subject to a structured uncertainty is determined by analyzing the feedback system in Figure 5.14. The nominal closed-loop system is assumed to be stable. Any unstable poles of this system are therefore caused by closing the loop through the perturbation and are the solutions of
\[
\det(\mathbf{I} - \mathbf{N}_{\Delta}(s)\Delta(s)) = 0. \tag{5.15}
\]
Stability robustness may be evaluated by determining the “size” of the smallest perturbation that results in a pole—a solution of (5.15)—with a non-negative real part. A perturbation that results in such a pole is termed a destabilizing perturbation.

The locus of the solutions of (5.15) is a continuous function of \( \Delta(s) \). Therefore, the smallest destabilizing perturbation has one or more poles on the imaginary axis. The “size” of the smallest destabilizing perturbation is defined as follows:

\[
\inf_{\omega} \left\{ \min_{\Delta(j\omega) \in \Delta} (\sigma(\Delta(j\omega)) \text{ such that } \det(\mathbf{I} - \mathbf{N}_{\Delta}(j\omega)\Delta(j\omega)) = 0) \right\}. \tag{5.16}
\]

Note that \( \Delta(j\omega) \) in (5.16) is any perturbation (with the appropriate block structure) that places a pole at a specific point \( j\omega \) on the imaginary axis. The maximum singular value is a measure of the size of this perturbation. The minimization over all appropriate perturbations results in the size of the smallest perturbation that places a pole at \( j\omega \). The minimization over frequency then yields the size of the smallest perturbation that places a pole anywhere on the imaginary axis, that is, the size of the smallest destabilizing perturbation. A system in standard form is robustly stable if and only if the smallest destabilizing perturbation is greater than 1 (the infinity norm of the largest admissible perturbation):

\[
\inf_{\omega} \left\{ \min_{\Delta(j\omega) \in \Delta} (\sigma(\Delta(j\omega)) \text{ such that } \det(\mathbf{I} - \mathbf{N}_{\Delta}(j\omega)\Delta(j\omega)) = 0) \right\} > 1. \tag{5.17}
\]

Unfortunately, finding the size of the smallest destabilizing perturbation using (5.16) is not a trivial matter. In fact, this problem is intractable in all but the simplest of cases. Therefore, bounds on the size of the smallest destabilizing perturbation are developed. Before generating these bounds, the stability robustness condition (5.17) is put in a form that is more useful for both application and computation.