To demonstrate this bound, first note that if \( \mu_\Delta(N) = 0 \), the bound is valid since \( \bar{\sigma}(N) \geq 0 \). Now, when \( \mu_\Delta(N) \neq 0 \), the definition of \( \mu_\Delta(N) \) states that

\[
\frac{1}{\mu_\Delta(N)} = \min_{\Delta \in \mathfrak{D}} [\bar{\sigma}(\Delta) \text{ such that } \det(I \vDash N\Delta) = 0].
\] (5.23)

Let \( \tilde{\Delta} \) be the value of \( \Delta \in \mathfrak{D} \) that yields the minimum in (5.23). The fact that the determinant in (5.23) equals zero implies that for some nonzero vector \( v \),

\[
(I \vDash N\tilde{\Delta})v = 0;
\]

\[
v = \sqrt{\bar{\sigma}(N\Delta)}v.
\] (5.24)

Taking the vector 2-norm of this equation, we have

\[
\|v\|_2 = \|N\tilde{\Delta}v\|_2.
\]

The gain of the matrix in this expression can be bounded by the maximum singular value:

\[
\|v\|_2 \leq \bar{\sigma}(N\tilde{\Delta})\|v\|_2;
\]

\[
\|v\|_2 \leq \bar{\sigma}(N)\bar{\sigma}(\tilde{\Delta})\|v\|_2.
\]

Dividing by \( \|v\|_2 \) and \( \bar{\sigma}(\tilde{\Delta}) \) yields the result given in (5.22):

\[
\mu_\Delta(N) = \frac{1}{\bar{\sigma}(\tilde{\Delta})} \leq \bar{\sigma}(N).
\]

In summary, the structured singular value of a matrix is bounded from above by the maximum singular value.

The bound in (5.22) is easy to compute but tends to be overly conservative; that is, the structured singular value can be appreciably less than the maximum singular value. Additional bounds can be generated by returning to the system interpretation of the SSV and considering the block diagrams in Figure 5.17.

The SSV of a transfer function \( N(s) \) is the inverse of the smallest perturbation that, when placed in the feedback loop, yields a closed-loop pole located at \( s \). The closed-loop poles of this system are not changed by the inclusion (as shown in Figure 5.17b) of the diagonal scaling matrices \( \mathfrak{D}_L(s) \) and \( \mathfrak{D}_R(s) \) and their inverses:

\[
\mathfrak{D}_L(s) = \begin{bmatrix}
\begin{array}{ccc}
\frac{d_1(s)}{I_{n_1}} & 0 & \cdots & 0 \\
0 & \frac{d_2(s)}{I_{n_2}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{d_m(s)}{I_{n_m}}
\end{array}
\end{bmatrix}
\]

(5.25a)

\[
\mathfrak{D}_R(s) = \begin{bmatrix}
\begin{array}{ccc}
\frac{d_1(s)}{I_{n_1}} & 0 & \cdots & 0 \\
0 & \frac{d_2(s)}{I_{n_2}} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{d_m(s)}{I_{n_m}}
\end{array}
\end{bmatrix}
\]

(5.25b)

The uncertainty blocks have the dimensions \( \Delta_i(s) \in \mathbb{C}^{n_i \times n_i} \), and the identity matrices have the dimensions \( I_{n_i} \in \mathbb{R}^{n_i \times n_i} \). These dimensions match up with the perturbation blocks to yield