ture inherent in this problem. Two of the more efficient of these algorithms are described in [6] and [9]. A very readable treatment of the basics of these algorithms can be found in [10], page 119.

The upper bound in (5.27) has been found to lie close to the true SSV in a number of applications. This bound is an equality for perturbations consisting of three or fewer complex blocks, typically within 5% of the true SSV for larger numbers of complex blocks, and rarely worse than within 15% of the true SSV. The exception is when some of the perturbations are real.

The bound in (5.27) may be overly conservative for mixed real and complex perturbations, although even for these mixed perturbations, (5.27) often yields acceptable results. The extension of (5.27) to the case of mixed real and complex perturbations is treated in [5, 6].

A bound similar to (5.27) can be developed for the case where repeated perturbations are present. Repeated perturbations occur when multiple system parameters are dependent on a single external factor. For example, the Mach number (speed) of an aircraft influences both the lift produced by a given angle of attack and the torque produced by a given angle of attack. Therefore, the two parameters in the model relating lift and moment to angle of attack are related. These parameters can be modeled as nominal values plus scaled versions of the same perturbation. An extension of (5.27) to the case of repeated perturbations is given in [7].

**Lower Bounds** The structured singular value is bounded from below by the spectral radius of $N_s$:

$$
\mu_3(N) \geq \rho(N_s) \tag{5.28}
$$

where $N_s$ is generated from $N$ by the addition of zero columns and/or rows. These zero columns and rows are added to make $N_s$ square and to make the perturbation blocks square, and have no effect on the structured singular value. The spectral radius of a matrix is defined as the largest of the eigenvalue magnitudes:

$$
\rho(N) = \max_i |\lambda_i(N)|.
$$

The following example is used to illustrate how $N_s$ is generated and to demonstrate that

$$
\mu_3(N) = \mu_3(N_s).
$$

Note that $\Delta_s$ is the set of block diagonal perturbations with appropriately sized square blocks.

**Example 5.11** Let $N \in \mathbb{C}^{2 \times 3}$, and $\Delta \in \mathbb{C}^{3 \times 2}$. The block structure of $\Delta$ is defined by

$$
\Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix},
$$

where $\Delta_1 \in \mathbb{C}^{2 \times 1}$, and $\Delta_2 \in \mathbb{C}^{1 \times 2}$. The determinant used in computing $\mu_3(N)$ is

$$
\det(I + NA) = \det \left( \begin{bmatrix} 1 & 0 & \Delta_1 N_{11} & \Delta_1 N_{12} & \Delta_1 N_{13} \\ 0 & 1 & \Delta_1 N_{21} & \Delta_1 N_{22} & \Delta_1 N_{23} \\ \end{bmatrix} \begin{bmatrix} \Delta_{11} & 0 \\ \Delta_{21} & 0 \\ 0 & \Delta_{32} \end{bmatrix} \right)
$$

$$
= \det \left( \begin{bmatrix} 1 & \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} & \end{bmatrix} \right).
$$