specifications for this value of \( K \). The nominal system is not required to meet the performance specifications when applying the SSV performance robustness test, but stability of the nominal system is required for application of this test. Better tracking can be obtained by increasing the feedback gain beyond 50. The system is seen to remain stable and meets the performance specifications for all admissible perturbations when \( K = 200 \). The design in this particular example could be accomplished using more traditional methods due to the simplicity of the system, but the example still serves to illustrate the application of the SSV to performance robustness analysis.

**Example 5.14**

We are given the vertical-takeoff-and-landing remotely piloted vehicle of Example 5.7. A block diagram of the pitch and yaw control system for this vehicle is given in Figure 5.13a. A reasonable performance goal is that the pitch and yaw errors be smaller than \( 1^\circ \), or about 5\% of the largest commanded values. Note that the gain requirement is derived by comparing the allowable error to the largest commanded input expected (about \( 20^\circ \)). The commanded values are assumed to be slowly varying; specifically, they are band-limited to less than 1 rad/sec. These specifications can be translated to a bound on the perturbed closed-loop transfer function:

\[
\sigma(H(j\omega)) \leq \begin{cases} 
0.05 & \omega \leq 1 \\
\infty & \omega > 1 
\end{cases}
\]

Note that the infinity in this bound indicates that the performance above 1 rad/sec is unimportant. The performance goal is normalized to 1 by the use of the following weighting function:

\[
W(j\omega) = \begin{cases} 
20 & \omega \leq 1 \\
0 & \omega > 1 
\end{cases}
\]

A rational approximation of this weighting function is

\[
W(s) = \frac{20}{(s + 1)^2} = \frac{20}{s^2 + 2s + 1}.
\]

The magnitude of the weighting function and its rational approximation are shown in Figure 5.22a. A second-order weighting function is used to achieve a rapid roll-off at high frequencies (the specifications call for an infinitely rapid roll-off). Appending this transfer function to the system and adding the performance block yields the block diagram in Figure 5.22b. Note that the feedback gains are given in Example 5.7. This system was shown to be robustly stable in Example 5.7, but no information was generated on performance. Nominal performance is evaluated by numerically computing the maximum singular value of the nominal closed-loop transfer function:

\[
\|N_{yw}(s)\|_{\infty} = W(s)\|\left(I - C_y(sI - A + B_uKC_m)^{-1}B_uKK_r\right)\|
\]

The principle gains for this transfer function are plotted in Figure 5.23. The infinity norm of the nominal system is 0.81, which indicates that the nominal system achieves the performance specifications. The total nominal closed-loop transfer function is