The estimate of the torque, given the orthogonal portion of the current measurement is

\[
\hat{T}_0[m_0 - \hat{m}_0(m_2)] = \frac{[R_T(0) - 0.41R_T(1)][m_0 - \hat{m}_0(m_2)]}{R_T(0) + \sigma_o^2 + 0.41^2R_T(0) + 0.41^2\sigma_o^2 - 0.82R_T(1)}
\]

\[
= 0.40[m_0 - \hat{m}_0(m_2)]
\]

Combining these results yields the optimal linear estimate of the current torque:

\[
\hat{T}_0(m_{-1}, m_0) = 0.41m_{-1} + 0.40(m_0 - 0.41m_{-1}) = 0.25m_{-1} + 0.40m_0.
\]

Note that this result is identical to the optimal estimate obtained previously.

The derivation of the estimator in this example was more complex than the direct derivation presented in Subsection 7.1.1. The advantage of updating an existing estimate is that it does not require recomputing all of the estimator gains for the data used in the existing estimate. This ability to sequentially process data is a significant practical advantage when processing long data strings in real time.

To summarize, the linear optimal estimate of the state, given two data vectors, can be computed sequentially. The state is initially estimated using the first data vector. The state is then estimated using the portion of the second data vector that is not predictable from the first data vector. The final state estimate is obtained by summing these two estimates. This procedure is reasonably intuitive when thinking of the unpredictable portion of the second data vector as new information. The summing of the two estimates can then be thought of as correcting the original estimate based on this new information.

### 7.2 The Kalman Filter

The Kalman filter estimates the state of a plant given a set of known inputs and a set of measurements. The plant is described by the state model:

\[
x(t) = Ax(t) + B_u u(t) + B_w w(t);
\]

\[
m(t) = C_m x(t) + v(t),
\]

which is driven by both a known, deterministic input \( u(t) \), and an unknown random input \( w(t) \) called the plant noise. The measurements from the plant \( m(t) \) are corrupted by a random measurement noise \( v(t) \). The plant and measurement noises are assumed to be white noise vectors with the spectral densities \( S_w \) and \( S_v \), respectively:

\[
E[w(t)w^T(t + \tau)] = S_w \delta(\tau); \tag{7.14a}
\]

\[
E[v(t)v^T(t + \tau)] = S_v \delta(\tau). \tag{7.14b}
\]

\[\text{The plant and measurement noises can be combined into a single disturbance input. Separating these inputs as in (7.13) matches the majority of the literature, and simplifies the discussion of the noise properties and the resulting filter performance.}\]