rial tends to be application-specific and is beyond the scope of this book. System identification is also a large field of study [4, 5] and is beyond the scope of this book. This subsection includes some general properties of mathematical models that make them suitable for use in Kalman filtering.

The plant model must be driven by noise and should include as states, or linear combinations of the states, all of the quantities that the filter is required to estimate. Additionally, the measurements, which are the outputs of this model, should be linear combinations of the states. As in all engineering applications, the model should be as simple as possible while still describing the relevant plant behavior. The modeling of the plant may be one of the most difficult tasks facing the Kalman filter designer.

**The Measurement Noise Spectral Density Matrix** The measurement noise spectral density matrix may be given by the sensor manufacturer, determined theoretically, or determined empirically. These methods of generating spectral density matrices are illustrated in the following examples.

### EXAMPLE 7.4

An accelerometer is being used to estimate the vibration of a rotating machine. The manufacturer of the accelerometer provides information on the error bounds of the sensor,

\[ |\text{Error}| \leq 0.001 \frac{\text{m}}{\text{sec}^2}, \]

and the sensor bandwidth:

\[ \text{Bandwidth} = 100 \text{ Hz}. \]

Assuming a uniform distribution of the error within the bounds (a reasonable assumption), the error variance is found to be

\[ \sigma^2 = \frac{0.001^2}{3} \frac{\text{m}^2}{\text{sec}^4}. \]

The measurement noise can be approximated by white noise provided the bandwidth of this noise is large compared to the plant bandwidth. In this case, the noise can be assumed to be white provided that all of the significant vibrational frequencies of the rotating machine are much less than 100 rad/sec. The measurement noise spectral density then equals the variance divided by twice the bandwidth:

\[ S_v = \frac{0.001^2}{600} = 1.67 \times 10^{-9} \frac{\text{m}^2}{\text{sec}^2 \cdot \text{Hz}}. \]

### EXAMPLE 7.5

A Kalman filter is used to estimate the range and the radial velocity of an aircraft given noisy range measurements provided by a fixed radar. The measurement noise is caused primarily by thermal electrons at the front end of the receiver. Careful modeling and a detailed theoretical analysis of this thermal noise yields the spectral density of a radar range estimate:

\[ S_v = \frac{c^2}{16B^3(S/N)}. \]