In summary, the suboptimal (or optimal) $\mathcal{H}_\infty$ output feedback control law is the $\mathcal{H}_\infty$ suboptimal (or optimal) estimate of the full information control. The controller is therefore designed in two stages: a full information controller is synthesized, and an output estimator is synthesized. The final output feedback controller is generated by combining these two components. This controller has a structure similar to the LQG controller, but technically violates the separation principle, since the estimator design depends on the full information controller design.

### 10.2 Finite-Time Control

The suboptimal $\mathcal{H}_\infty$ output estimation problem specified by (10.6) through (10.8) has a solution if the Riccati equation

\[
\dot{Q}_m(t) = -Q_m(t)(A + \gamma^{-2}B_w^T B_w P(t))^{-1} \left[ A + \gamma^{-2}B_w^T B_w P(t)Q_m(t) \right] + \frac{1}{\gamma^2} B_w^T Q_m(t) C_m^T C_m Q_m(t) - \gamma^{-2} P(t) B_w^T B_w P(t) Q_m(t),
\]

has a solution, given the initial condition $Q_m(0) = 0$. The suboptimal estimator is

\[
\dot{x}(t) = [A + \gamma^{-2}B_w B_w^T P(t)] \dot{x}(t) + B_w u(t) + G(t)[m(t) - C_m \dot{x}(t)];
\]

\[
\dot{u}(t) = -B_w^T P(t) \dot{x}(t),
\]

and the gain is

\[
G(t) = Q_m(t) C_m^T.
\]

This estimator contains two known inputs: $u(t)$ which enters through $B_w$, and $\gamma^{-2}B_w^T P(t) \dot{x}(t)$ which enters through $B_w$. The second input is the worst-case disturbance encountered during full information controller optimization. This estimator can then be described as estimating the full information control in the presence of the worst case disturbance.

This estimator can be used as a feedback controller. Substituting for the gain (10.10) in the estimator and setting $u(t) = \dot{u}(t)$ yields the controller:

\[
\dot{x}(t) = [A + \gamma^{-2}B_w B_w^T P(t) - B_w B_w^T P(t) - Q_m(t) C_m^T C_m] \dot{x}(t) + Q_m(t) C_m^T m(t);
\]

\[
\dot{u}(t) = -B_w^T P(t) \dot{x}(t).
\]

Solutions to the Riccati equation (10.9) and the full information Riccati equation (9.13) are sufficient to guarantee the existence of this suboptimal $\mathcal{H}_\infty$ controller.

### 10.2.1 An Alternative Estimator Riccati Equation

The generation of the $\mathcal{H}_\infty$ output feedback controller (as given above) proceeds by synthesizing the full information control and then estimating this control. This process results in solving the full information Riccati equation for the plant. In addition, an estimator Riccati equation for a modified plant is solved. This Riccati solution can be related to the Riccati solution associated with reference output estimation for the original plant. Developing this correspondence produces a symmetric (the symmetry is between the control and estimation Riccati equations) pair of Riccati equations that can be solved to

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1. The full information control estimator includes the worst case disturbance as a known input. This fact will be discussed in Section 10.2.