EE 3140: Engineering Electromagnetics

Hour Test #1  Feb. 15, 07  Name: KEY  (last)  (first)

Grade: __________

This test contains ten questions and a Bonus question. Each question is worth ten points total. Full credit is given to the correct answer. No partial credit is given if work is not shown neatly. Please work carefully.

Q1: Consider a tiny particle on a string moving up and down expressed mathematically as shown below:

\[ y(x,t) = 10 \sin \left( \frac{10\pi t - 40\pi x}{10} \right) \text{ m} \]

Obtain an expression for the vertical velocity of the particle as a function of time.

\[ \frac{dy}{dt} = 10 \cos \left( \frac{10\pi t - 40\pi x}{10} \right) \frac{10\pi}{10} = 100 \pi \cos \left( \frac{10\pi t - 40\pi x}{10} \right) \text{ m/s} \]

Q2: An EM wave traveling in seawater was observed to have an amplitude of 100 V/m at a depth of 10m and an amplitude of 80 V/m at a depth of 80m. What is the attenuation constant of sea water. Sketch the amplitude vs. depth in sea water from 0 to 100m.

\[ E(x) = E_0 e^{-\alpha x} \]

\[ 100 = E_0 e^{-\alpha 10} \]
\[ 80 = E_0 e^{-\alpha 80} \]
\[ \frac{100}{80} = e^{\alpha 10} \]
\[ \alpha = \frac{\ln(100/80)}{10} = 0.0032 \text{ N/p/m} \]

Q3: a) Find

\[ (1-j)^{1/2} = \left( 1.414 e^{-j 45^\circ} \right)^{1/2} \]

\[ E_0 = 100 e^{-j 10} e^{-j 10} = 100 e^{-1.189} \approx 103.2 \]

\[ \pm 1.09 - j0.455 \]

\[ \pm 1.189 \pm 22.5 \]

b) If \( z_1 = 5\angle 60^\circ \) and \( z_2 = 2\angle 5^\circ \) find \( z_1 z_2 \)
Q4: Express the following complex scalars as instantaneous time sinusoidal functions

5) \[ I = 6 + j8 \quad \Leftrightarrow \quad 6 \cos(\omega t) - 8 \sin(\omega t) \]

\[ \sqrt{6^2 + 8^2} \cos(\omega t + 53.13^\circ) = 10 \cos(\omega t + 53.13^\circ) \]

\[ 10 \cos(\omega t + 0.927) \]

5) \[ \tilde{V} = 2 e^{j \pi/6} \quad \Leftrightarrow \quad 2 \cos(\omega t + \pi/6) \]

Q5: Given vectors \( \mathbf{A} = \hat{x} 2 - \hat{y} 3 + \hat{z} 4 \) and \( \mathbf{B} = \hat{x} 2 - \hat{y} 4 \) find \( \mathbf{A} \cdot \mathbf{B} \) and \( \mathbf{A} \times \mathbf{B} \)

5) \[ \mathbf{A} \cdot \mathbf{B} = 4 + 12 = 16 \]

\[ \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -3 & 4 \\ 2 & -4 & 0 \end{vmatrix} = \hat{x}(16) + \hat{y} 8 - \hat{z} 2 \]

Q6: Find the angle between \( \mathbf{A} \) and \( \mathbf{B} \). Use \( \mathbf{A} \) and \( \mathbf{B} \) given in Q5.

\[ \cos \theta_{AB} = \frac{|\mathbf{A} \cdot \mathbf{B}|}{|\mathbf{A}| |\mathbf{B}|} = \frac{16}{\sqrt{29} \sqrt{20}} = 0.66 \]

\[ \theta_{AB} = 48.26^\circ \]
Q7: Given \( \mathbf{E} = \hat{x} 10 - \hat{y} xyz + \hat{z} x^2 y^3 z^4 \), find the line integral for the length shown.

\[
\mathbf{E} \cdot d\mathbf{c} = \int_0^5 10 \, dx = 10 \left[ x \right]_0^5 = 50
\]

Q8: If \( V = x y^2 z^3 \) find \( \nabla V \) and \( \nabla \cdot \nabla V \)

\[
\nabla V = \hat{x} \frac{\partial}{\partial x} (x y^2 z^3) + \hat{y} \frac{\partial}{\partial y} (x y^2 z^3) + \hat{z} \frac{\partial}{\partial z} (x y^2 z^3)
\]

\[
= \hat{x} y^2 z^3 + \hat{y} 2x y z^3 + \hat{z} 3x y^2 z^2
\]

\[
\nabla \cdot \nabla V = \left( \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \right) \cdot (2x y^2 z^3 + \hat{y} 2x y z^3 + \hat{z} 3x y^2 z^2)
\]

\[
= 0 + 2x y^2 z^3 + 6x y^2 z^2
\]

Q9: Write two equations in both differential and integral forms (from Maxwell’s equations) that all static electric fields need to satisfy in free space in a region free of sources. How these equations change if the medium is not free space but a medium like sand with \( \varepsilon_r = 4 \)?

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\[
\nabla \cdot \mathbf{D} = \sigma_v \quad \text{or} \quad \nabla \cdot \mathbf{E} = \frac{\sigma_v}{\varepsilon_0} = 0
\]

\[
\nabla \times \mathbf{E} = 0
\]

4

\[
\text{Change medium} \quad \nabla \cdot \mathbf{E} = \frac{\sigma_v}{\varepsilon_0 \varepsilon_r} \leq 0
\]

\[
\nabla \times \mathbf{E} = 0
\]
Q10: Find the electric field vector, $\mathbf{E}(y, t)$ that is always there with a magnetic field vector, $\mathbf{H}(y, t) = \hat{x} \ 0.3 \ \cos{(\omega t + ky)} \ \text{A/m}$ using the appropriate Maxwell's equation. The medium is free space and free of sources.

\[ \mathbf{H} \text{ in } x \text{- direction} \quad \mathbf{B}_0 = \frac{\mathbf{E}_0}{c} = \omega c \mathbf{H}_0 \]

\[ \mathbf{E}_0 = c \mu_0 \mathbf{H}_0 = \mu_0 c (0.3) \cos{(\omega t + ky)} \]

\[ \nabla \times \mathbf{H} = \mathbf{k} \text{ direction } \Rightarrow \text{ wave is } -\hat{y} \]

\[ -\hat{z} \times \hat{x} = -\hat{y} \]

\[ \mathbf{E}(y, t) = -\hat{z} \ 113 \cos{(\omega t + ky)} \ \text{V/m} \]

\[ \nabla \times \mathbf{E} = \mathbf{0} \quad \nabla \cdot \mathbf{E} = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial}{\partial t} \mathbf{E} \]

**BONUS QUESTION:**

Show mathematically that $\nabla \cdot (\nabla \times \mathbf{A}) = 0$ for any vector $\mathbf{A}$. Explain in one sentence why the divergence of a curl of a vector is always zero.

**OR**

Write a lyrical poem on the Del operator.

\[ \nabla \cdot (\nabla \times \mathbf{A}) = \nabla \cdot \left[ \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{array} \right] = \nabla \cdot \left[ \hat{r} \left( \frac{\partial \mathbf{A}_x}{\partial y} - \frac{\partial \mathbf{A}_y}{\partial x} \right) - \hat{\mathbf{z}} \right] \]