Ongoing List of Topics:

- URL: http://www.ece.mtu.edu/faculty/bamork/EE5223/index.htm
- Term Project - last few proj/teams being firmed up and getting moving.
  - Follow timeline, see posting on web page (posted in week 5)
  - Weeks 7 thru 9 - develop formal outline w/complete reference list
- Homework set 9 to be completed by Wed after break (Wed Mar 16th)
  - Typo on Prob#1: circle radius is 2, i.e. \( r = 4 \).
  - Prob#2 c) – what is actual reach of relay? Think about it.
- More on Transmission Line protection schemes - read Ch.13
  - Pilot schemes (general designation)
    - Blocking vs. unblocking; permissive vs. nonpermissive
    - Underreaching vs. overreaching
    - Directional comparison vs. phase comparison
    - Transfer Trip
    - Traveling wave methods (require wide-bandwidth current transducer, such as "Rogowski Coil")?
- Protection fundamentals for 87T, cont’d - b) relay settings are used to compensate for pri voltage ratio and CT ratios. c) Mismatch problems - due to being forced to use less than full CT ratio, and having Pri and Sec CTs with different accuracy levels. Differential slope of trip characteristic can be 10%, 15%, 25% to allow for mismatch. Print out XFR.pdf!

- Bus Diff, - Cap Bank, - Motor, - Gen, ...
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\[ \frac{63}{26} \times \text{op} \delta \text{GT} \]

\[ \frac{63}{26} \text{Trip} \]

\[ \frac{63}{26} \text{Trip} \]

\[ \frac{63}{26} \text{HIL} \]

\[ \frac{63}{26} \text{LIL} \]

OR-ing of "dry" contacts.

63 = sudden pressure
26 = oil temp
49 = winding not spot
71 = low oil level

Mavsic
and

\[ V_c = V_{hi} - (jX_{cc})I_{rated} \]

\[ = 1.0/0^\circ - (j0.06)(1.0/36.87^\circ) \]

\[ = 1.0 - (j0.06)(0.8 - j0.6) = 0.952 - j0.064 \]

\[ = 0.954/38.85^\circ \text{ per unit} \]

b. As shown in Figure 4.20(b),

\[ I_{cc} = \frac{V_c}{X_{cc}} = 1.0/0^\circ = 12.5 \text{ per unit} \]

Under rated current conditions (part a), the 0.08 per-unit voltage drop across the transformer leakage reactance causes the voltage at the low-voltage terminals to be 0.954 per unit. Also, under three-phase short-circuit conditions (part b), the fault current is 12.5 times the rated transformer current. This example illustrates a compromise in the design or specification of transformer leakage reactance. A low value is desired to minimize voltage drops, but a high value is desired to limit fault currents. Typical transformer leakage reactances are given in Table A.2 in the Appendix.

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**SECTION 4.6**

**THREE-WINDING TRANSFORMERS**

Figure 4.21(a) shows a basic single-phase three-winding transformer. The ideal transformer relations for a two-winding transformer, (4.1.3) and (4.1.14), can easily be extended to obtain corresponding relations for an ideal three-winding transformer. In actual units, these relations are:

\[ N_1 I_1 = N_2 I_2 + N_3 I_3 \]  \hspace{1cm} (4.6.1)

\[ \frac{E_1}{N_1} = \frac{E_2}{N_2} = \frac{E_3}{N_3} \]  \hspace{1cm} (4.6.2)

where \( I_1 \) enters the dotted terminal, \( I_2 \) and \( I_3 \) leave dotted terminals, and \( E_1, E_2, \) and \( E_3 \) have their + polarities at dotted terminals. In per-unit, (4.6.1) and (4.6.2) are:

\[ I_{1pu} = I_{2pu} + I_{3pu} \]  \hspace{1cm} (4.6.3)

\[ E_{1pu} = E_{2pu} = E_{3pu} \]  \hspace{1cm} (4.6.4)

where a common \( S_{pu} \) is selected for all three windings, and voltage bases are selected in proportion to the rated voltages of the windings. These two per-unit relations are satisfied by the per-unit equivalent circuit shown in Figure 4.21.
4.21(b). Also, external series impedance and shunt admittance branches are included in the practical three-winding transformer circuit shown in Figure 4.21(c). The shunt admittance branch, a core loss resistor in parallel with a magnetizing inductor, can be evaluated from an open-circuit test. Also, when one winding is left open, the three-winding transformer behaves as a two-winding transformer, and standard short-circuit tests can be used to evaluate per-unit leakage impedances, which are defined as follows:

\[ Z_{12} = \text{per-unit leakage impedance measured from winding 1, with winding 2 shorted and winding 3 open} \]

\[ Z_{13} = \text{per-unit leakage impedance measured from winding 1, with winding 3 shorted and winding 2 open} \]

\[ Z_{23} = \text{per-unit leakage impedance measured from winding 2, with winding 3 shorted and winding 1 open} \]

From Figure 4.21(c), with winding 2 shorted and winding 3 open, the leakage impedance measured from winding 1 is, neglecting the shunt admittance branch,

\[ Z_{12} = Z_1 + Z_3 \]  
(4.6.5)

Similarly,

\[ Z_{13} = Z_1 + Z_3 \]  
(4.6.6)

and

\[ Z_{23} = Z_2 + Z_3 \]  
(4.6.7)

Solving (4.6.5)–(4.6.7),

\[ Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23}) \]  
(4.6.8)

\[ Z_2 = \frac{1}{2}(Z_{12} + Z_{23} - Z_{13}) \]  
(4.6.9)

\[ Z_3 = \frac{1}{2}(Z_{13} + Z_{23} - Z_{12}) \]  
(4.6.10)

Equations (4.6.8)–(4.6.10) can be used to evaluate the per-unit series impedances \( Z_1, Z_2, \) and \( Z_3 \) of the three-winding transformer equivalent circuit from the per-unit leakage impedances \( Z_{12}, Z_{13}, \) and \( Z_{23} \), which, in turn, are determined from short-circuit tests.

Note that each of the windings on a three-winding transformer may have a different kVA rating. If the leakage impedances from short-circuit tests are expressed in per-unit based on winding ratings, they must first be converted to per-unit on a common \( S_{base} \) before they are used in (4.6.8)–(4.6.10).

**Example 4.10**

Three-winding single-phase transformer: per-unit impedances

The ratings of a single-phase three-winding transformer are:

winding 1: 300 MVA, 13.8 kV

winding 2: 300 MVA, 199.2 kV

winding 3: 50 MVA, 19.92 kV

The leakage reactances, from short-circuit tests, are:

\[ X_{12} = 0.10 \text{ per unit on a 300-MVA, 13.8-kV base} \]

\[ X_{13} = 0.16 \text{ per unit on a 50-MVA, 13.8-kV base} \]

\[ X_{23} = 0.14 \text{ per unit on a 50-MVA, 199.2-kV base} \]

Winding resistances and exciting current are neglected. Calculate the impedances of the per-unit equivalent circuit using a base of 300 MVA and 13.8 kV for terminal 1.

**Solution**

\( S_{base} = 300 \text{ MVA} \) is the same for all three terminals. Also, the specified voltage base for terminal 1 is \( V_{base1} = 13.8 \text{ kV} \). The base voltages for terminals 2 and 3 are then \( V_{base2} = 199.2 \text{ kV} \) and \( V_{base3} = 19.92 \text{ kV} \), which are the rated voltages of these windings. From the data given, \( X_{12} = 0.10 \) per unit was measured from terminal 1 using the same base values as those specified for the circuit. But \( X_{13} = 0.16 \) and \( X_{23} = 0.14 \) per unit on a 50-MVA base are first converted to the 300-MVA circuit base.

\[ X_{12} = (0.10) \left( \frac{300}{50} \right) = 0.96 \text{ per unit} \]

\[ X_{13} = (0.16) \left( \frac{300}{50} \right) = 0.84 \text{ per unit} \]

Then, from (4.6.8)–(4.6.10),

\[ X_1 = \frac{1}{2}(0.10 + 0.96 - 0.84) = 0.11 \text{ per unit} \]

\[ X_2 = \frac{1}{2}(0.10 + 0.84 - 0.96) = -0.01 \text{ per unit} \]

\[ X_3 = \frac{1}{2}(0.96 + 0.84 - 0.10) = 0.85 \text{ per unit} \]

The per-unit equivalent circuit of this three-winding transformer is shown in Figure 4.22. Note that \( X_1, X_2, \) and \( X_3 \) are not leakage reactances, but instead are equivalent reactances derived from the leakage reactances. Leakage reactances are always positive.

Note also that the node where the three equivalent circuit reactances...
are connected does not correspond to any physical location within the transformer. Rather, it is simply part of the equivalent circuit representation.

Three identical single-phase three-winding transformers can be connected to form a three-phase bank. Figure 4.23 shows the general per-unit sequence networks of a three-phase three-winding transformer. Instead of labeling the windings 1, 2, and 3, as was done for the single-phase transformer, the letters H, M, and X are used to denote the high-, medium-, and low-voltage windings, respectively. By convention, a common S max is selected for the H, M, and X terminals, and voltage bases V max, V max, and V maxx are selected in proportion to the rated line-to-line voltages of the transformer.

For the general zero-sequence network, Figure 4.23(a), the connection between terminals H and H' depends on how the high-voltage windings are connected, as follows:

1. Solidly grounded Y—Short H to H'.
2. Grounded Y through $Z_y$—Connect ($Z_y$) from H to H'.
3. Ungrounded Y—Leave H–H' open as shown.
4. $\Delta$—Short H' to the reference bus.

Terminals X–X' and M–M' are connected in a similar manner.

Example 4.11
Three-winding three-phase transformer: per-unit sequence networks

Three transformers, each identical to that described in Example 4.10, are connected as a three-phase bank in order to feed power from a 900-MVA, 13.8-kV generator to a 345-kV transmission line and to a 34.5-kV distribution line. The transformer windings are connected as follows:

- 13.8-kV windings (X): $\Delta$, to generator
- 345-kV windings (H): solidly grounded Y, to 345-kV line
- 19.92-kV windings (M): grounded Y through $Z_y = 0.10 \Omega$, to 34.5-kV line

Figure 4.24
Per-unit sequence networks for Example 4.11

(a) Zero sequence
(b) Positive sequence
(c) Negative sequence
The positive-sequence voltages and currents of the high- and medium-voltage Y windings lead the corresponding quantities of the low-voltage Δ winding by 30°. Draw the per-unit sequence networks, using a three-phase base of 900 MVA and 13.8 kV for terminal X.

Solution

The per-unit sequence networks are shown in Figure 4.24. Since \( V_{\text{base}} = 13.8 \) kV is the rated line-to-line voltage of terminal X, \( V_{\text{base}} = \sqrt{3}(19.92) = 34.5 \) kV, which is the rated line-to-line voltage of terminal M. The base impedance of the medium-voltage terminal is then

\[
Z_{\text{base}} = \frac{(34.5)^2}{900} = 1.3225 \ \Omega
\]

Therefore, the per-unit neutral impedance is

\[
Z_n = \frac{1.010}{1.3225} = 0.76561 \ \text{per unit}
\]

and \((3Z_n) = 0.2268\) is connected from terminal M to M' in the per-unit zero-sequence network. Since the high-voltage windings have a solidly grounded neutral, H to H' is shorted in the zero-sequence network. Also, phase-shifting transformers are included in the positive- and negative-sequence networks.

SECTION 4.7

AUTOTRANSFORMERS

A single-phase two-winding transformer is shown in Figure 4.28(a) with two separate windings, which is the usual two-winding transformer; the same transformer is shown in Figure 4.25(b) with the two windings connected in series, which is called an autotransformer. For the usual transformer (Figure 4.25(a)) the two windings are coupled magnetically via the mutual core flux. For the autotransformer (Figure 4.25(b)) the windings are both electrically and magnetically coupled. The autotransformer has smaller per-unit leakage impedances than the usual transformer; this results in both smaller series-voltage drops (an advantage) and higher short-circuit currents (a disadvantage). The autotransformer also has lower per-unit losses (higher efficiency), lower exciting current, and lower cost if the turns ratio is not too large. The electrical connection of the windings, however, allows transient overvoltages to pass through the autotransformer more easily.

EXAMPLE 4.12

The single-phase two-winding 20-kVA, 480/120-volt transformer of Example 4.3 is connected as an autotransformer, as in Figure 4.25(b), where winding 1 is the 120-volt winding. For this autotransformer, determine (a) the voltage ratings \( E_a \) and \( E_b \) of the low- and high-voltage terminals, (b) the kVA rating, and (c) the per-unit leakage impedance.

Solution

a. Since the 120-volt winding is connected to the low-voltage terminal, \( E_a = 120 \) volts. When \( E_a = E_1 = 120 \) volts is applied to the low-voltage terminal, \( E_3 = 480 \) volts is induced across the 480-volt winding, neglecting the voltage drop across the leakage impedance. Therefore, \( E_a = E_1 + E_3 = 120 + 480 = 600 \) volts.

b. As a normal two-winding transformer rated 20 kVA, the rated current of the 480-volt winding is \( I_a = I_3 = 20,000/480 = 41.667 \) A. As an autotransformer, the 480-volt winding can carry the same current. Therefore, the kVA rating \( S_a = E_a I_a = (600)(41.667) = 25 \) kVA. Note also that when \( I_a = I_1 = 41.667 \) A, a current \( I_3 = \frac{480}{120} = 4.0 \) A is induced in the 120-volt winding. Therefore, \( I_3 = I_1 + I_3 = 208.3 \) A (neglecting exciting current) and \( S_a = E_a I_a = (120)(208.3) = 25 \) kVA, which is the same rating as calculated for the high-voltage terminal.

c. From Example 4.3, the leakage impedance is 0.0729/78.12° per unit as a normal, two-winding transformer. As an autotransformer, the leakage impedance is also the same as for the normal transformer, since the core and windings are the same for both (only the external winding connections are different). But the base impedances are different. For the high-voltage terminal, using (4.3.4),

\[
Z_{\text{base}} = \frac{(480)^2}{20,000} = 11.52 \ \Omega \text{ as a normal transformer}
\]

\[
Z_{\text{base}} = \frac{(600)^2}{25,000} = 14.4 \ \Omega \text{ as an autotransformer}
\]

Therefore, using (4.3.10),

\[
Z_{\text{base}} = \frac{0.0729/78.12°}{(11.52) / 14.4} = 0.0583/78.12° \text{ per unit}
\]

For this example, the rating is 25 kVA, 120/600 volts as an autotransformer versus 20 kVA, 120/480 volts as a normal transformer. The autotransformer has both a larger kVA rating and a larger voltage ratio for the same cost.
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"Thin Fault"

Note: Typical scheme used by Consumer's Energy.