Transformer Modeling for Simulation of Low Frequency Transients

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Introduction

- Large number of core designs
- Some of transformer parameters are both nonlinear and frequency dependent
- Physical attributes whose behavior may need to be correctly represented
  - core and coil configurations
  - self- and mutual inductances between coils
  - leakage fluxes
  - skin effect and proximity effect in coils
  - magnetic core saturation and hysteresis
  - eddy current losses in core
  - capacitive effects
- Aim of this presentation
Transformer Models

- Matrix representation
  - BCTRAN model
- Saturable Transformer Component (STC)
- Topology-based models
  - Duality based models
  - Geometric models

Matrix representation (BCTRAN model)

- Branch impedance matrix of a multi-phase multi-winding transformer
  - Steady state equations: \( [V] = [Z] [I] \)
  - Transient equations: \( [v] = [R] [i] + [L] \left[ \frac{di}{dt} \right] \)

\([R]\) and \(\omega[L]\) are the real and the imaginary part of \([Z]\), whose elements can be derived from excitation tests.

- The approach includes phase-to-phase couplings, models terminal characteristics, but does not consider differences in core or winding topology.
Transformer Models

Saturable Transformer Component (STC model)

Star-circuit representation of single-phase N-winding transformers

Models derived using duality

Core design                     Equivalent circuit
Duality-derived model for a single-phase shell-form transformer
### MODEL EQUATIONS CHARACTERISTICS

<table>
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<th>MODEL</th>
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</table>
| **Matrix Representation (BCTRAN model)** | $v = [R]i + [L]di/dt$ | • These models include all phase-to-phase coupling and terminal characteristics.  
• Only linear models can be represented.  
• Excitation may be attached externally at the terminals in the form of non-linear elements.  
• They are reasonable accurate for frequencies below 1 kHz. |
| **Saturable Transformer Component (STC model)** | $v = (L)^{-1}([L]^{-1}iL + [dL/di])$ | • It cannot be used for more than 3 windings.  
• The magnetising inductance is connected to the star point.  
• Numerical instability can be produced with 3-winding models. |
| **Topology-based models** | • Duality-based models: They are derived using a circuit-based approach without a mathematical description  
• Geometric models: $v = [R]i + [di/dt]$ | • Duality-based models include the effects of saturation in each individual leg of the core, interphase magnetic coupling, and leakage effects.  
• The mathematical formulation of geometric models is based on the magnetic equations and their coupling to the electrical equations, which is made taking into account the core topology. Models differ from each other in the way in which the magnetic equations are derived. |

### Nonlinear and Frequency-Dependent Parameters

- Some transformer parameters are nonlinear and/or frequency-dependent due to:
  - saturation
  - hysteresis
  - eddy currents

- Saturation and hysteresis introduce distortion in waveforms
- Hysteresis and eddy currents originate losses
- Saturation is predominant in power transformers, but eddy current and hysteresis effects can play an important role in some transients
Nonlinear and Frequency-Dependent Parameters

- Modeling of iron cores
  - Iron core behavior represented by a relationship between the magnetic flux density $B$ and the magnetic field intensity $H$
  - Each magnetic field value is related to an infinity of possible magnetizations depending on the history of the sample
  - To characterize the material behavior fully, a model has to be able to plot
    - major and minor hysteresis loops
      (minor loops can be symmetric or asymmetric)

Magnetization curves and hysteresis loops
Modeling of iron cores

- Equivalent circuit for representing a nonlinear inductor

- Hysteresis loops have a negligible influence on the magnitude of the magnetizing current
  - Hysteresis losses can have some influence on some transients; the residual flux has a major influence on the magnitude of inrush currents

- The saturation characteristic can be modeled by a piecewise linear inductance with two slopes, except in some cases, e.g. ferroresonance

Eddy current effects

- Excitation losses are mostly iron-core losses
  - hysteresis and eddy current losses
  - they cannot be separated
  - hysteresis losses are much smaller than eddy current losses

- Eddy current models for
  - transformer windings
  - iron laminated cores
Eddy current effects

Models for windings

\[ R_0 - R_1 \left( \frac{1}{L_1} + R_1 \right) \cdots R_N \left( \frac{1}{L_N} + R_N \right) \]

Series Foster equivalent circuit

Eddy current effects

Models for iron laminated cores

\[ R_1 \left( \frac{1}{L_1} + R_1 \right) \cdots R_N \left( \frac{1}{L_N} + R_N \right) \]

Standard Cauer equivalent

\[ R_1 - R_2 \left( \frac{1}{L_2} + R_2 \right) \cdots R_N \left( \frac{1}{L_N} + R_N \right) \]

Dual Cauer equivalent
Transformer Models

Matrix representation (BCTRAN model)

BCTRAN model for a three-phase three-legged stacked core transformer

Parameter Determination

- Data usually available for any power transformer
  - power rating
  - voltage rating
  - excitation current
  - excitation voltage
  - excitation losses
  - short-circuit current
  - short-circuit voltage
  - short-circuit losses
  - saturation curve
  - capacitances between terminals and between windings

- Excitation and short-circuit currents, voltages and losses must be provided from both direct and homopolar measurements
Parameter Determination

- An accurate representation for three-phase core transformers should be based on the core topology, include eddy current effects and saturation/hysteresis representation
- A very careful representation and calculation of leakage inductances is usually required
- Coil-capacitances have to be included for an accurate simulation of some transients
- Since no standard procedures have been developed, a parameter estimation seems to be required regardless of the selected model
- Temperature influence should not be neglected

Modeling - Case 1

Three-legged stacked-core transformer
Cross section of core and winding assembly
Modeling - Case 1

Three-legged stacked-core transformer
Duality-based equivalent circuit

Modeling - Case 1

Three-legged stacked-core transformer
ATP implementation
Modeling - Case 1

Three-leg stacked-core transformer
Excitation currents

Modeling - Case 1

Three-leg stacked-core transformer
Inrush currents
**Ferroresonance - Case 2**

![Ferroresonance Diagram](image)

**Magnetic Saturation - Case 2**

![Magnetic Saturation Curves](image)

8-Segment Curve vs. 2-Segment Curve
Steady-State Excitation - Case 2

Ferroresonance - Case 2A
Ferroresonance - Case 2B

Ferroresonance: 15uF, 2-Segment Magnetization Curve

Effect of Mag Curve Representation

Ferroresonant Voltage: Red(2A): 8-Seg; Blue(2B): 2-Seg @ 1.0A; Green: 2-Seg @ 1.5A
Effect of Mag Curve Representation

Conclusions

- There is no agreement on the most adequate model.
- Modeling difficulties:
  - great variety of core designs
  - nonlinear and frequency dependent parameters
  - inadequacy for acquisition and determination of some transformer parameters
- Several modeling levels could be considered since not all parameters have the same influence on all transient phenomena.