EE 380 - Homework - Suggested completion is Monday, Jan 17th. More problems will be assigned and collected on Wednesday, so it would be good to get these out of the way.

Note - These 2 problems have been almost completed as in-class examples. Refer to your notes as you work through them. Focus on the basic concepts and try to develop engineering insights (avoid approaching this as a procedure if you can).

H2.1) A single-phase autotransformer has an input voltage of 1380 Volts and supplies a 277-Volt 15-kW load of PF = 0.8 lag. Assuming that the voltage at the load has a reference angle of zero degrees,

a) Draw the complete circuit, including source, transformer, and load. Label all voltages and currents. Show polarity markings on the transformer windings.
b) Determine the phasor value of the current flowing into the load.
c) Determine the phasor value of the currents in the 2 windings of the autotransformer, and specify the required voltage and current ratings for each of the windings.
d) What is that phase angle of the source voltage?
e) Calculate the volt-amp advantage of this particular transformer.
f) Explain what the volt-amp advantage is, by contrasting the performance and cost of this autotransformer with an equivalent 2-winding transformer.

H2.2) A single-phase transformer has low-voltage rating of 230 Volts. Using the simplified equivalent circuit discussed in class, its parameters, referred to the low-voltage side, are: $R_c = 1050 \Omega$, $X_m = 110 \Omega$, $R_{EQ} = 0.0445 \Omega$, and $X_{EQ} = 0.0645 \Omega$. A load is connected to the low-voltage side. Source voltage is controlled in each case so that the load voltage is 230 /0° Volts.

a) Draw the complete circuit, including source, transformer, and load. Label all voltages and currents. Show polarity markings on the transformer windings.
b) A 15-kVA load of PF = 0.8 LAG is connected. Calculate the phasor voltage $V_2'$ and draw the voltage phasor diagram for this case showing $V_2$, $V_2'$, and the voltage drops across $R_{EQ}$ and $X_{EQ}$. Calculate the voltage regulation for this load. Calculate the "load losses" of the transformer (i.e. the $I^2R$ losses in the transformer windings).
c) Repeat for a load of unity power factor.
d) Repeat for PF = 0.8 LEAD.
e) What observations can you make about expected voltage regulation for loads of varying power factor? What is the significance of a positive voltage regulation? What is the significance of a negative voltage regulation? What power factor is desirable if we wish to minimize the system losses due to the flow of load current?
a) $\bar{I}_{\text{load}} = \frac{15,000}{277 \times 0.8} \frac{1}{\cos 10.8}$

$= 67.69 \angle -36.87^\circ \text{ A}$

b) $I_1 = I_{\text{load}} \times \frac{277}{1380} = 13.58 \text{ A}$

$\bar{I}_1 = 13.58 \angle -36.87^\circ \text{ A}$

KCL: $\bar{I}_2 = I_{\text{load}} - \bar{I}_1 = 54.1 \angle -36.87^\circ \text{ A}$

Winding 1 (Also called "series" winding)

$V_{\text{RATED}} = 1380 - 277 = 1103 \text{ V}$

$I_{\text{RATED}} = 13.58 \text{ A}$

Winding 2 (Also called "common" winding)

$V_{\text{RATED}} = 277 \text{ V}$

$I_{\text{RATED}} = 54.1 \text{ A}$
d) $I_1$, $I_2$, and $I_L$ are all in phase, in accordance with polarity markings. This also determines that $V_1$, $V_2$, $V_{load}$, and $V_{source}$ are in phase. i.e. $\angle V_{source} = 0^\circ$.

e) If used as a 2-winding transformer,

\[ \begin{array}{c}
13.6 \text{ A} \\
1103 \text{ V} \\
\end{array} \quad \begin{array}{c}
54.1 \text{ A} \\
277 \text{ V} \\
\end{array} \]

$VA_{pri} = (1103 \times 13.6) = 15,000 \leq$ Reassuring

$VA_{sec} = (277 \times 54.1) = 15,000 \leq$ to see they're equal.

If connected as an auto,

\[ \begin{array}{c}
13.6 \text{ A} \\
1380 \text{ V} \\
\end{array} \quad \begin{array}{c}
67.7 \text{ A} \\
277 \text{ V} \\
\end{array} \]

$VA = (1380 \times 13.6) = (277 \times 67.7) = 18,768 \text{ VA}$
The VOLT-AMP advantage is the ratio of the VA rating as auto divided by VA rating as 2-winding.

\[ \text{VA Advantage} = \frac{18,768}{15,000} = 1.25 \]

This means that it can transform 15.7% more volt-amps when connected as an autotransformer.

Using cook-book equations, we should get same result.

From class notes,

\[ \frac{V_L}{V_H - V_L} + 1 = \frac{277}{1380 - 277} + 1 = 1.25 \]

or

\[ \frac{V_H I_1}{V_1 I_1} = \frac{(1380)(13.6)}{(1103)(13.6)} = 1.25 \]

f) We can use 2-winding XFRM of only 80% the required VA rating, connect as auto, and be all set. Since cost is approx proportional to VA size, we can save 20% off the cost by using an auto.
b) \[ I_{\text{Load}} = \frac{15,000}{230} \angle \cos 0.8 \]

\[ = 65.3 \angle -36.87^\circ \text{ A} \]

KVL: \[ V_2' = V_2 + I_{\text{Load}} (R_{\text{Eq}} + jX_{\text{Eq}}) \]

\[ = 230 (\angle 0^\circ + (65.3 \angle -36.87)(0.0443) + (65.3 \angle -36.87)(-0.0645)) \]

\[ = 234.85 \angle +0.4^\circ \text{ V} \]

\[ VR = \frac{V_{\text{NL}} - V_{\text{FL}}}{V_{\text{FL}}} = \frac{234.85 - 230}{230} = +2.1\% \]
(Side note in preparation for parts c & d): In every case \( P_{\text{out}} = P_{\text{load}} \).
If we assume that \( P \) is the same for all 3 cases, from case 1,

\[
P = 15,000 \times \text{PF} = 12,000 \text{ W}
\]

Transformer load losses are \( I^2R \)

\[
= (65.3)^2 (0.0443) = 188.9 \text{ W}
\]

c) For unity PF, \( I_{\text{load}} = \frac{12,000}{230} = 52.17 \text{ A} \)

\[
I_{\text{load}} = 52.17 / 0^\circ \text{ A}
\]

\[
V_2' = 230 / 0^\circ + (52.17 / 0^\circ)(0.0443 + (52.17 / 0^\circ)(0.0645)) = 232.3 / +0.83^\circ \text{ V}
\]
c) (cont'd)

\[ VR = \frac{232.8 - 230}{230} = +0.96\% \]

Winding losses:

\[ P_{\text{loss}} = (52.17)^2 (0.0443) \]

\[ = 120 \text{ W} \]

d) \(PF = 0.8\) leading

\[ I_{\text{load}} = 65.3 (+36.87^\circ) \text{ A} \]

\[ V_2' = 230 \angle 0^\circ + (65.3 (+36.87))(0.0443) + (65.3 (+36.87))(0.0645) \]

\[ = 229.8 (+1.27^\circ) \text{ V} \]

\[ VR = \frac{229.8 - 230}{230} = -0.017\% \]

e)

*** Note! Leading PF loads actually increase the load voltage, i.e. the load voltage decreases when the load is disconnected. This may seem counter-intuitive. However, let's look at the voltage phasor diagram:
If we take this to an extreme, for a pure capacitive load,

\[ I_{\text{load}} = \pm 90^\circ \]

We can think of this as the capacitive current causing a "negative voltage drop" across the inductor \( jX_{\text{EQ}} \).

\[ P_{\text{req}} = \frac{(65.3)^2(0.0443)}{188.9 \text{ W}} \]

\( \approx \) same as for 0.8 lag.

\( \approx \) Max efficiency when PF = unity.