Topics for Today:

- **Announcements**
  - Term Project: Keep cranking. Monday of finals week ok?
  - Software: Matlab, ASPEN
  - Office hrs: 2:00-3:00pm, Mon, Wed, Fri
  - Office: EERC 623. Phone: 906.487.2857
  - All solutions thru Ch.15 are posted.

Chapter 9 - Load Flow exercise
- Bus voltage, etc.

Chapter 13 - Power system operation, AGC, economic dispatch
- Paralleling of Generators
- No-load set point, droop characteristics
- Per unit use of droop characteristic for many generators
- Optimization methods - LaGrange multipliers
- Economic dispatch
The purpose of this laboratory is to gain insight into the operation of a utility power grid. A simple 5-bus system is to be analyzed using a Newton-Raphson power flow program. Operating problems are to be identified and various means of correcting them shall be explored.

The basic system is given as:

\[ \text{MVA}_{\text{base}} = 100 \text{ MVA} \]

The network diagram is shown with buses labeled 1 to 5 and load data summarized in a table. The generation and load data, along with initial values, are provided. The diagram includes a note on voltage limits exceeding, suggesting a change to a PQ bus with a revised Qmax.

The table details generation and load data, while the network diagram outlines the connections between buses. The network is described as "Flat" start, indicating a simplified approach.

The electrical parameters, such as line length, resistance, reactance, and charging power factor, are listed in a table. The maximum line flow is calculated as 1.0 per unit.

Mathematical note: At rated (i.e. 1.0 p.u.) voltage, the expression given is: \[ \frac{R_{\text{base}} + X_{\text{base}}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 0.205 \text{ p.u.} \]
Droop Characteristic (Refer to AEC Notes)

- Move droop char. upward.
- $\Delta P_m = -\frac{1}{A} \Delta F$
- If set-point isn't changed, $R = \frac{\Delta F}{\Delta P_m}$

Can track $f$ either in Hz or p.u.

Pis typically tracked in p.u. (although text examples are in Hz)
Two scenarios:

1) Controlled change in $P_{setEO}$.
   - Change $F_{noLoad}$, let gov "do its thing."

2) Uncontrolled event: line trip
   - $P_{out}$ suddenly decreases,
     $f \uparrow$

\[ \Delta P_m = -\frac{1}{R} \Delta f \]

(See p. 2a, AGC Notes)
Ex: 8.5

\[ Se_i = -\frac{\Delta P_i}{\Delta Q_i} \]

\[ R_i = -\frac{\Delta Q_i}{\Delta P_i} \]

Go thru calculations, will resume here on Thursday (Lecture 26).

m
The efficiency of the generator is thus

\[
\eta = \frac{P_{\text{m}}}{P_{\text{in}}} \times 100\% \\
= \frac{34.1 \text{ kW}}{36.6 \text{ kW}} \times 100\% \\
= 93.2\%
\]

(d) The input torque to this generator is given by the equation

\[
P_{\text{in}} = \tau_{\text{app}} \omega_m
\]

so

\[
\tau_{\text{app}} = \frac{P_{\text{in}}}{\omega_m} = \frac{36.6 \text{ kW}}{125.7 \text{ rad/s}} = 291.2 \text{ N} \cdot \text{m}
\]

The induced countertorque is given by

\[
P_{\text{conv}} = \tau_{\text{ind}} \omega_m
\]

so

\[
\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{34.1 \text{ kW}}{125.7 \text{ rad/s}} = 271.3 \text{ N} \cdot \text{m}
\]

(e) The voltage regulation of a generator is defined as

\[
VR = \frac{V_{\text{nl}} - V_0}{V_0} \times 100\% 
\]

By this definition, the voltage regulation for the lagging, unity, and leading power-factor cases are

\begin{enumerate}
  \item Lagging case: \( VR = \frac{480 \text{ V} - 410 \text{ V}}{410 \text{ V}} \times 100\% = 17.1\% \)
  \item Unity case: \( VR = \frac{480 \text{ V} - 468 \text{ V}}{468 \text{ V}} \times 100\% = 2.6\% \)
  \item Leading case: \( VR = \frac{480 \text{ V} - 535 \text{ V}}{535 \text{ V}} \times 100\% = -10.3\% \)
\end{enumerate}

In Example 8-3, lagging loads resulted in a drop in terminal voltage, unity-power-factor loads caused little effect on \( V_r \), and leading loads resulted in an increase in terminal voltage.

8-9 PARALLEL OPERATION OF AC GENERATORS

In today's world, an isolated synchronous generator supplying its own load independently of other generators is very rare. Such a situation is found in only a few out-of-the-way applications such as emergency generators. For all usual generator applications, there is more than one generator operating in parallel to supply the power demanded by the loads. An extreme example of this situation is the U.S. power grid, in which literally thousands of generators share the load on the system.

Why are synchronous generators operated in parallel? There are several major advantages to such operation:

1. Several generators can supply a bigger load than one machine by itself.
2. Having many generators increases the reliability of the power system, since the failure of any one of them does not cause a total power loss to the load.
3. Having many generators operating in parallel allows one or more of them to be removed for shutdown and preventive maintenance.
4. If only one generator is used and it is not operating at near full load, then it will be relatively inefficient. But with several smaller machines it is possible to operate only a fraction of them. The ones that do operate are operating near full load and thus more efficiently.

This section explores the requirements for paralleling ac generators and then looks at the behavior of synchronous generators operated in parallel.

The Conditions Required for Paralleling

Figure 8-25 shows a synchronous generator \( G_1 \) supplying power to a load, with another generator \( G_2 \) about to be paralleled with \( G_1 \) by closing the switch \( S_1 \). What conditions must be met before the switch can be closed and the two generators connected?

If the switch is closed arbitrarily at some moment, the generators are liable to be severely damaged, and the load may lose power. If the voltages are not exactly the same in each conductor being tied together, there will be a very large

![Figure 8-25](image-url)

A generator being paralleled with a running power system.
current flow when the switch is closed. To avoid this problem, each of the three phases must have exactly the same voltage magnitude and phase angle as the conductor to which it is connected. In other words, the voltage in phase $a$ must be exactly the same as the voltage in phase $a'$, and so forth for phases $b$-$b'$ and $c$-$c'$.

To achieve this match, the following paralleling conditions must be met:

1. The rms line voltages of the two generators must be equal.
2. The two generators must have the same phase sequence.
3. The phase angles of the two $a$ phases must be equal.
4. The frequency of the new generator, called the oncoming generator, must be slightly higher than the frequency of the running system.

These paralleling conditions require some explanation. Condition 1 is obvious—in order for two sets of voltages to be identical, they must of course have the same rms magnitude of voltage. The voltage in phases $a$ and $a'$ will be completely identical at all times if both their phases and their magnitudes are the same, which explains condition 3.

Condition 2 ensures that the sequence in which the phase voltages peak in the two generators is the same. If the phase sequence is different (as shown in Fig. 8-26a), then even though one pair of voltages (the $a$ phases) are in phase, the other two pairs of voltages are $120^\circ$ out of phase. If the generators were connected in this manner, there would be no problem with phase $a$, but huge currents would flow in phases $b$ and $c$, damaging both machines. To correct a phase sequence problem, simply swap the connections on any two of the three phases on one of the machines.

If the frequencies of the generators are not very nearly equal when they are connected together, large power transients will occur until the generators stabilize at a common frequency. The frequencies of the two machines must be very nearly equal, but they cannot be exactly equal. They must differ by a small amount so that the phase angles of the oncoming machine will change slowly with respect to the phase angles of the running system. In that way, the angles between the voltages can be observed and switch $S_1$ can be closed when the systems are exactly in phase.

**The General Procedure for Paralleling Generators**

Suppose that generator $G_2$ is to be connected to the running system shown in Fig. 8-26. To accomplish the paralleling, the following steps should be taken.

First, using voltmeters, the field current of the oncoming generator should be adjusted until its terminal voltage is equal to the line voltage of the running system.

Second, the phase sequence of the oncoming generator must be compared to the phase sequence of the running system. The phase sequence can be checked in a number of different ways. One way is to alternately connect a small induction motor to the terminals of each of the two generators. If the motor rotates in the same direction each time, then the phase sequence is the same for both generators. If the motor rotates in opposite directions, then the phase sequences differ, and two of the conductors on the incoming generator must be reversed.

Another way to check the phase sequence is the three-light-bulb method. In this approach, three light bulbs are stretched across the open terminals of the switch connecting the generator to the system as shown in Fig. 8-26b. As the phase changes between the two systems, the light bulbs first get bright (large phase difference) and then get dim (small phase difference). If all three bulbs get bright and dark together, then the systems have the same phase sequence. If the bulbs brighten in succession, then the systems have the opposite phase sequence, and one of the sequences must be reversed.

Next, the frequency of the oncoming generator is adjusted to be slightly higher than the frequency of the running system. This is done first by watching a frequency meter until the frequencies are close and then by observing changes in
phase between the systems. The oncoming generator is adjusted to a slightly higher frequency so that when it is connected, it will come on the line supplying power as a generator, instead of consuming it as a motor would (this point will be explained later).

Once the frequencies are very nearly equal, the voltages in the two systems will change phase with respect to each other very slowly. The phase changes are observed, and when the phases angles are equal, the switch connecting the two systems together is shut.

How can one tell when the two systems are finally in phase? A simple way is to watch the three light bulbs described above in connection with the discussion of phase sequence. When the three light bulbs all go out, the voltage difference across them is zero and the systems are in phase. This simple scheme works, but it is not very accurate. A better approach is to employ a synchroscope. A synchroscope is a meter that measures the difference in phase angle between the two phases of the two systems. The face of a synchroscope is shown in Fig. 8-27. The dial shows the phase difference between the two phases, with 0° (meaning in phase) at the top and 180° at the bottom. Since the frequencies of the two systems are slightly different, the phase angle on the meter changes slowly. If the oncoming generator or system is faster than the running system (the desired situation), then the phase angle advances and the synchroscope needle rotates clockwise. If the oncoming machine is slower, the needle rotates counterclockwise. When the synchroscope needle is in the vertical position, the voltages are in phase, and the switch can be shut to connect the systems.

Notice, though, that a synchroscope checks the relationships on only one phase. It gives no information about phase sequence.

In large generators belonging to power systems, this whole process of paralleling a new generator to the line is automated, and a computer does this job. For smaller generators, though, the operator manually goes through the paralleling steps just described.

**Frequency-Power and Voltage-Reactive Power Characteristics of a Synchronous Generator**

All generators are driven by a prime mover, which is the generator's source of mechanical power. The most common type of prime mover is a steam turbine, but other types include diesel engines, gas turbines, water turbines, and even wind turbines.

Regardless of the original power source, all prime movers tend to behave in a similar fashion—as the power drawn from them increases, the speed at which they turn decreases. The decrease in speed is in general nonlinear, but some form of governor mechanism is usually included to make the decrease in speed linear with an increase in power demand.

Whatever governor mechanism is present on a prime mover, it will always be adjusted to provide a slight drooping characteristic with increasing load. The speed droop (SD) of a prime mover is defined by the equation

\[
SD = \frac{n_{nl} - n_f}{n_f} \times 100\% \tag{8-27}
\]

where \(n_{nl}\) is the no-load prime-mover speed and \(n_f\) is the full-load prime-mover speed. Most generators have a speed droop of 2 to 4 percent, as defined in Eq. (8-27). In addition, most governors have some type of set point adjustment to allow the no-load speed of the turbine to be varied. A typical speed-versus-power plot is shown in Fig. 8-28.

Since the shaft speed is related to the resulting electrical frequency by Eq. (7-14),

\[
f_e = \frac{n_m P}{120} \tag{7-14}
\]

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**FIGURE 8-27**  
A synchroscope.

**FIGURE 8-28**  
(a) The speed-versus-power curve for a typical prime mover. (b) The resulting frequency-versus-power curve for the generator.
the power output of a synchronous generator is related to its frequency. An example plot of frequency versus power is shown in Fig. 8-28b. Frequency-power characteristics of this sort play an essential role in the parallel operation of synchronous generators.

The relationship between frequency and power can be described quantitatively by the equation

\[
P = s_p (f_{nl} - f_{sys})
\]  

(8-28)

where

- \(P\) = power output of generator
- \(f_{nl}\) = no-load frequency of generator
- \(f_{sys}\) = operating frequency of system
- \(s_p\) = slope of curve, \(\text{kW/Hz}\) or \(\text{MW/Hz}\)

A similar relationship can be derived for the reactive power \(Q\) and terminal voltage \(V_T\). As previously seen, when a lagging load is added to a synchronous generator, its terminal voltage drops. Likewise, when a leading load is added to a synchronous generator, its terminal voltage increases. It is possible to make a plot of terminal voltage versus reactive power, and such a plot has a drooping characteristic like the one shown in Fig. 8-29. This characteristic is not necessarily linear, but many generator voltage regulators include a feature to make it so. The characteristic curve can be moved up and down by changing the no-load terminal voltage set point on the voltage regulator. As with the frequency-power characteristic, this curve plays an important role in the parallel operation of synchronous generators.

The relationship between the terminal voltage and reactive power can be expressed by an equation similar to the frequency-power relationship [Eq. (8-28)]

\[
V_T, V
\]

\[
V_{Tnl}, V_{Tsys}
\]

- \(Q\), \(\text{kVAR consumed}\)
- \(Q\), \(\text{kVAR supplied}\)

**FIGURE 8-29**
The terminal voltage \((V_T)\)-versus-reactive power \((Q)\) curve for a synchronous generator.

if the voltage regulator produces an output that is linear with changes in reactive power.

It is important to realize that when a single generator is operating alone, the real power \(P\) and reactive power \(Q\) supplied by the generator will be the amount demanded by the load attached to the generator—the \(P\) and \(Q\) supplied cannot be controlled by the generator’s controls. Therefore, for any given real power, the governor set points control the generator’s operating frequency \(f_\nu\), and for any given reactive power, the field current controls the generator’s terminal voltage \(V_T\).

**Example 8-4.** Figure 8-30 shows a generator supplying a load. A second load is to be connected in parallel with the first one. The generator has a no-load frequency of 61.0 Hz and a slope \(s_p\) of 1 MW/Hz. Load 1 consumes a real power of 1000 kW at 0.8 PF lagging, while load 2 consumes a real power of 800 kW at 0.707 PF lagging.

(a) Before the switch is closed, what is the operating frequency of the system?
(b) After load 2 is connected, what is the operating frequency of the system?
(c) After load 2 is connected, what action could an operator take to restore the system frequency to 60 Hz?

**Solution.** This problem states that the slope of the generator’s characteristic is 1 MW/Hz and that its no-load frequency is 61 Hz. Therefore, the power produced by the generator is given by

\[
P = s_p (f_{nl} - f_{sys})
\]  

(8-28)

so

\[
f_{sys} = f_{nl} - \frac{P}{s_p}
\]

(a) The initial system frequency is given by

\[
f_{sys} = f_{nl} - \frac{1000 \text{ kW}}{1 \text{ MW/Hz}}
\]

\[
= 61 \text{ Hz} - 1.0 \text{ Hz}
\]

\[
= 60 \text{ Hz}
\]

**FIGURE 8-30**
The power system in Example 8-4.
(b) After load 2 is connected,

\[
f_{syn} = f_n - \frac{1800 \text{ kW}}{1 \text{ MW/Hz}}
\]

\[
= 61 \text{ Hz} - 1.8 \text{ Hz}
\]

\[
= 59.2 \text{ Hz}
\]

(c) After the load is connected, the system frequency falls to 59.2 Hz. To restore the system to its proper operating frequency, the operator should increase the governor no-load set points by 0.8 Hz to 61.8 Hz. This action will restore the system frequency to 60 Hz.

To summarize, when a generator is operating by itself supplying the system loads, then

1. The real and reactive power supplied by the generator will be the amount demanded by the attached load.
2. The governor set points of the generator will control the operating frequency of the power system.
3. The field current (or the field regulator set points) control the terminal voltage of the power system.

This is the situation found in isolated generators in remote field environments.

**Operation of Generators in Parallel with Large Power Systems**

When a synchronous generator is connected to a power system, the power system is often so large that *nothing* the operator of the generator does will have much of an effect on the power system. An example of this situation is the connection of a single generator to the U.S. power grid. The U.S. power grid is so large that no reasonable action on the part of the one generator can cause an observable change in overall grid frequency.

This idea is idealized in the concept of an infinite bus. An infinite bus is a power system so large that its voltage and frequency do not vary regardless of how much real and reactive power is drawn from or supplied to it. The power-frequency characteristic of such a system is shown in Fig. 8-31a, and the reactive power-voltage characteristic is shown in Fig. 8-31b.

To understand the behavior of a generator connected to such a large system, examine a system consisting of a generator and an infinite bus in parallel supplying a load. Assume that the generator's prime mover has a governor mechanism, but that the field is controlled manually by a resistor. It is easier to explain generator operation without considering an automatic field current regulator, so this discussion will ignore the slight differences caused by the field regulator when one is present. Such a system is shown in Fig. 8-32a.

When a generator is connected in parallel with another generator or a large system, the frequency and terminal voltage of all the machines must be the same.
Once the generator has been connected, what happens when its governor set points are increased? The effect of this increase is to shift the no-load frequency of the generator upward. Since the frequency of the system is unchanged (the frequency of an infinite bus cannot change), the power supplied by the generator increases. This is shown by the house diagram in Fig. 8-35a and by the phasor diagram in Fig. 8-35b. Notice in the phasor diagram that $E_A \sin \delta$ (which is proportional to the power supplied as long as $V_r$ is constant) has increased, while the magnitude of $E_A (= K \phi \omega)$ remains constant, since both the field current $I_F$ and the speed of rotation $\omega$ are unchanged. As the governor set points are further increased, the no-load frequency increases and the power supplied by the generator increases. As the power output increases, $E_A$ remains at constant magnitude while $E_A \sin \delta$ is further increased.

What happens in this system if the power output of the generator is increased until it exceeds the power consumed by the load? If this occurs, the extra power generated flows back into the infinite bus. The infinite bus, by definition, can supply or consume any amount of power without a change in frequency, so the extra power is consumed.

![Diagram](image)

**FIGURE 8-33**
The frequency-versus-power diagram at the moment just after paralleling.

![Diagram](image)

**FIGURE 8-34**
The frequency-versus-power diagram if the no-load frequency of the generator were slightly less than system frequency before paralleling.

![Diagram](image)

**FIGURE 8-35**
The effect of increasing the governor's set points on (a) the house diagram; (b) the phasor diagram.
After the real power of the generator has been adjusted to the desired value, the phasor diagram of the generator looks like Fig. 8-35b. Notice that at this time the generator is actually operating at a slightly leading power factor, so it is acting as a capacitor, supplying negative reactive power. Alternatively, the generator can be said to be consuming reactive power. How can the generator be adjusted so that it will supply some reactive power \( Q \) to the system? This can be done by adjusting the field current of the machine. To understand why this is true, it is necessary to consider the constraints on the generator's operation under these circumstances.

The first constraint on the generator is that the power must remain constant when \( I_e \) is changed. The power into a generator is given by the equation \( P_{in} = \tau_{app} \omega \omega' \). Now, the prime mover of a synchronous generator has a fixed torque-speed characteristic for any given governor setting. This curve changes only when the governor set points are changed. Since the generator is tied to an infinite bus, its speed cannot change. If the generator's speed does not change and the governor set points have not been changed, the power supplied by the generator must remain constant.

If the power supplied is constant as the field current is changed, then the distances proportional to the power in the phasor diagram (\( I_e \cos \theta \) and \( E_a \sin \delta \)) cannot change. When the field current is increased, the flux \( \phi \) increases, and therefore \( E_a = K \phi \sin \omega' \) increases. If \( E_a \) increases, but \( E_a \sin \delta \) must remain constant, then the phasor \( E_a \) must "slide" along the line of constant power, as shown in Fig. 8-36. Since \( V_b \) is constant, the angle of \( jX_b I_b \) changes as shown, and therefore the angle and magnitude of \( I_b \) change. Notice that as a result the distance proportional to \( Q (I_b \sin \theta) \) increases. In other words, increasing the field current in a synchronous generator operating in parallel with an infinite bus increases the reactive power output of the generator.

To summarize, when a generator is operating in parallel with an infinite bus:

1. The frequency and terminal voltage of the generator are controlled by the system to which it is connected.
2. The governor set points of the generator control the real power supplied by the generator to the system.
3. The field current in the generator controls the reactive power supplied by the generator to the system.

This situation is much the way real generators operate when connected to a very large power system.

**Operation of Generators in Parallel with Other Generators of the Same Size**

When a single generator operated alone, the real and reactive powers (\( P \) and \( Q \)) supplied by the generator were fixed, constrained to be equal to the power demanded by the load, and the frequency and terminal voltage were varied by the governor set points and the field current. When a generator operated in parallel with an infinite bus, the frequency and terminal voltage were constrained to be constant by the infinite bus, and the real and reactive powers were varied by the governor set points and the field current. What happens when a synchronous generator is connected in parallel not with an infinite bus, but rather with another generator of the same size? What will be the effect of changing governor set points and field currents?

If a generator is connected in parallel with another one of the same size, the resulting system is as shown in Fig. 8-37a. In this system, the basic constraint is that the sum of the real and reactive powers supplied by the two generators must equal the \( P \) and \( Q \) demanded by the load. The system frequency is not constrained to be constant, and neither is the power of a given generator constrained to be constant. The power-frequency diagram for such a system immediately after \( G_2 \) has been paralleled to the line is shown in Fig. 8-37b. Here, the total power \( P_{tot} \) (which is equal to \( P_{load} \)) is given by

\[
P_{tot} = P_{load} = P_{G1} + P_{G2}
\]

and the total reactive power is given by

\[
Q_{tot} = Q_{load} = Q_{G1} + Q_{G2}
\]

What happens if the governor set points of \( G_2 \) are increased? When the governor set points of \( G_2 \) are increased, the power-frequency curve of \( G_2 \) shifts upward, as shown in Fig. 8-37c. Remember, the total power supplied to the load must not change. At the original frequency \( f_1 \), the power supplied by \( G_1 \) and \( G_2 \) will now be larger than the load demand, so the system cannot continue to operate at the same frequency as before. In fact, there is only one frequency at which the sum of the powers out of the two generators is equal to \( P_{load} \). That frequency \( f_2 \) is higher than the original system operating frequency. At that frequency, \( G_2 \) supplies more power than before, and \( G_1 \) supplies less power than before.

Therefore, when two generators are operating together, an increase in governor set points on one of them

1. Increases the system frequency
2. Increases the power supplied by that generator, while reducing the power supplied by the other one

What happens if the field current of \( G_2 \) is increased? The resulting behavior is analogous to the real-power situation and is shown in Fig. 8-37d. When two generators are operating together and the field current of \( G_2 \) is increased.
1. The system terminal voltage is increased.
2. The reactive power $Q$ supplied by that generator is increased, while the reactive power supplied by the other generator is decreased.

If the slopes and no-load frequencies of the generator's speed droop (frequency-power) curves are known, then the powers supplied by each generator and the resulting system frequency can be determined quantitatively. Example 8-5 shows how this can be done.

Example 8-5. Figure 8-37 shows two generators supplying a load. Generator 1 has a no-load frequency of 61.5 Hz and a slope $s_p$ of 1 MW/Hz. Generator 2 has a no-load frequency of 61.0 Hz and a slope $s_p$ of 1 MW/Hz. The two generators are supplying a real load totaling 2.5 MW at 0.8 PF lagging. The resulting system power-frequency or house diagrams are shown in Fig. 8-38.

(a) At what frequency is this system operating, and how much power is supplied by each of the two generators?
(b) Suppose an additional 1-MW load were attached to this power system. What would the new system frequency be, and how much power would $G_1$ and $G_2$ supply now?
(c) With the system in the configuration described in part (b), what will the system frequency and generator powers be if the governor set points on $G_2$ are increased by 0.5 Hz?

**Solution.** The power produced by a synchronous generator with a given slope and no-load frequency is given by Eq. (8-28):

$$P_1 = s_p(f_{n1} - f_{sys})$$

and

$$P_2 = s_p(f_{n2} - f_{sys})$$

![FIGURE 8-37](image)

(a) A generator connected in parallel with another machine of the same size. (b) The corresponding house diagram at the moment generator 2 is paralleled with the system. (c) The effect of increasing generator 2's governor set points on the operation of the system. (d) The effect of increasing generator 2's field current on the operation of the system.

![FIGURE 8-38](image)

The house diagram for the system in Example 8-5.
Since the total power supplied by the generators must equal the power consumed by the loads,

\[ P_{\text{load}} = P_1 + P_2 \]

These equations can be used to answer all the questions asked.

(a) In the first case, both generators have a slope of 1 MW/Hz, and \( G_1 \) has a no-load frequency of 61.5 Hz, while \( G_2 \) has a no-load frequency of 61.0 Hz. The total load is 2.5 MW. Therefore, the system frequency can be found as follows:

\[ P_{\text{load}} = P_1 + P_2 = s_p(f_{\text{n1}} - f_{\text{sys}}) + s_p(f_{\text{n2}} - f_{\text{sys}}) \]

2.5 MW = (1 MW/Hz)(61.5 Hz - f_{\text{sys}}) + (1 MW/Hz)(61 Hz - f_{\text{sys}})

= 61.5 MW - (1 MW/Hz)(f_{\text{sys}}) + 61 MW - (1 MW/Hz)(f_{\text{sys}})

= 122.5 MW - (2 MW/Hz)(f_{\text{sys}})

(2 MW/Hz)(f_{\text{sys}}) = 120 MW

\[ f_{\text{sys}} = 60.0 \text{ Hz} \]

The resulting powers supplied by the two generators are

\[ P_1 = s_p(f_{\text{n1}} - f_{\text{sys}}) = (1 \text{ MW/Hz})(61.5 \text{ Hz} - 60.0 \text{ Hz}) = 1.5 \text{ MW} \]

and

\[ P_2 = s_p(f_{\text{n2}} - f_{\text{sys}}) = (1 \text{ MW/Hz})(61.0 \text{ Hz} - 60.0 \text{ Hz}) = 1 \text{ MW} \]

(b) When the load is increased by 1 MW, the total load becomes 3.5 MW. The new system frequency is now given by

\[ P_{\text{load}} = s_p(f_{\text{n1}} - f_{\text{sys}}) + s_p(f_{\text{n2}} - f_{\text{sys}}) \]

3.5 MW = (1 MW/Hz)(61.5 Hz - f_{\text{sys}}) + (1 MW/Hz)(61 Hz - f_{\text{sys}})

= 61.5 MW - (1 MW/Hz)(f_{\text{sys}}) + 61 MW - (1 MW/Hz)(f_{\text{sys}})

= 122.5 MW - (2 MW/Hz)(f_{\text{sys}})

(2 MW/Hz)(f_{\text{sys}}) = 119 MW

\[ f_{\text{sys}} = 59.5 \text{ Hz} \]

The resulting powers are

\[ P_1 = (1 \text{ MW/Hz})(61.5 \text{ Hz} - 59.5 \text{ Hz}) = 2.0 \text{ MW} \]

and

\[ P_2 = (1 \text{ MW/Hz})(61.0 \text{ Hz} - 59.5 \text{ Hz}) = 1.5 \text{ MW} \]

(c) If the no-load governor set points of \( G_2 \) are increased by 0.5 Hz, the new system frequency becomes

\[ P_{\text{load}} = s_p(f_{\text{n1}} - f_{\text{sys}}) + s_p(f_{\text{n2}} - f_{\text{sys}}) \]

3.5 MW = (1 MW/Hz)(61.5 Hz - f_{\text{sys}}) + (1 MW/Hz)(61 Hz - f_{\text{sys}})

= 123 MW - (2 MW/Hz)(f_{\text{sys}})

(2 MW/Hz)(f_{\text{sys}}) = 119.5 MW

\[ f_{\text{sys}} = 59.75 \text{ Hz} \]

The resulting powers are

\[ P_1 = P_2 = (1 \text{ MW/Hz})(61.5 \text{ Hz} - 59.75 \text{ Hz}) = 1.75 \text{ MW} \]

Notice that the system frequency rose, the power of \( G_2 \) rose, and the power of \( G_1 \) fell.

When two generators of similar size are operating in parallel, a change in the governor set points of one of them changes both the system frequency and the power sharing between them. It would normally be desired to adjust only one of these quantities at a time. How can the power sharing of the power system be adjusted independently of the system frequency, and vice versa?

The answer is very simple. An increase in governor set points on one generator increases that machine’s power and increases system frequency. A decrease in governor set points on the other generator decreases that machine’s power and decreases the system frequency. Therefore, to adjust power sharing without changing the system frequency, increase the governor set points of one generator and simultaneously decrease the governor set points of the other generator (see Fig. 8-39a). Similarly, to adjust the system frequency without changing the power sharing, simultaneously increase or decrease both governor set points (see Fig. 8-39b).

Reactive power and terminal voltage adjustments work in an analogous fashion. To shift the reactive power sharing without changing \( V_p \), simultaneously increase the field current on one generator and decrease the field current on the other (see Fig. 8-39c). To change the terminal voltage without affecting the reactive power sharing, simultaneously increase or decrease both field currents (see Fig. 8-39d).

To summarize, in the case of two generators operating together:

1. The system is constrained in that the total power supplied by the two generators together must equal the amount consumed by the load. Neither \( f_{\text{sys}} \) nor \( V_p \) is constrained to be constant.

2. To adjust the real power sharing between generators without changing \( f_{\text{sys}} \), simultaneously increase the governor set points on one generator while decreasing the governor set points on the other. The machine whose governor set point was increased will assume more of the load.
3. To adjust \( f_{\text{sys}} \) without changing the real power sharing, simultaneously increase or decrease both generators' governor set points.

4. To adjust the reactive power sharing between generators without changing \( V_R \), simultaneously increase the field current on one generator while decreasing the field current on the other. The machine whose field current was increased will assume more of the reactive load.

5. To adjust \( V_R \) without changing the reactive power sharing, simultaneously increase or decrease both generators' field currents.

It is very important that any synchronous generator intended to operate in parallel with other machines have a *drooping* frequency-power characteristic. If two generators have flat or nearly flat characteristics, then the power sharing between them can vary widely with only the tiniest changes in no-load speed. This problem is illustrated by Fig. 8-40. Notice that even very tiny changes in \( f_{\text{sys}} \) in one of the generators would cause wild shifts in power sharing. To ensure good control of power sharing between generators, they should have speed droops in the range of 2 to 5 percent.

### 8-10 SYNCHRONOUS GENERATOR TRANSIENTS

When the shaft torque applied to a generator or the output load on a generator changes suddenly, there is always a transient lasting for a finite period of time before the generator returns to steady state. For example, when a synchronous generator is paralleled with a running power system, it is initially turning faster and has a higher frequency than the power system does. Once it is paralleled, there is a transient period before the generator steadies down on the line and runs at line frequency while supplying a small amount of power to the load.