Topics for Today:

- Announcements
  - Expanded Term Project outline (i.e. Table of Contents + List of references (suggest about half a dozen to start with)) by beginning of this week.
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - ASPEN software - arranging to run off of MTU server via internet.
  - Office: EERC 623. Phone: 906.487.2857
  - Recommended problems & all solutions: Ch.7 solns posted.

- Chapter 7 - Network Equations, Admittance Approaches
  - How's your linear algebra? Time to make use of it...
  - Basic strategy for building up $[Y]$ for whole network
  - Quick recap of xfmrs and lines.
  - Generators
  - Example of building $[Y]$ for 4-bus system.
  - Network Reduction (Kron Reduction)
  - Solution of matrix equations (system of linear equations)
  - Upcoming homework - intro to Matlab, matrices, equations.
\[ I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a} \] (7.3)

FIGURE 7.1
Circuits illustrating the equivalence of sources when \( I_s = E_s/Z_a \) and \( Y_a = 1/Z_a \).
FIGURE 7.3
Single-line diagram of the four-bus system of Example 7.1. Reference node is not shown.

FIGURE 7.4
Reactance diagram for Fig. 7.3. Node (0) is reference, reactances and voltages are in per unit.
\[ I_N = \frac{V_{TH}}{Z_{TH}} \]

\[ I_{IN} = V_{TH}Y_N \]

\[ [Z] = [Y]^{-1} \]

\[ \begin{bmatrix} Y \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & -1 & -1 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

**FIGURE 7.5**
Per-unit admittance diagram for Fig. 7.4 with current sources replacing voltage sources. The names \( a \) to \( g \) correspond to the subscripts of branch voltages and currents.

Equation (7.9) applies to each of the other five branches. By setting \( m \) and \( n \) in equations equal to the node numbers at the ends of the individual branches of Fig. 7.5, we obtain:

\[ \begin{align*}
3 & \quad [1]Y_e \\
3 & \quad [1]Y_b \\
3 & \quad [1]Y_c \\
4 & \quad [1]Y_f \\
2 & \quad [1]Y_d \\
2 & \quad [1]Y_e \\
2 & \quad [1]Y_b \\
2 & \quad [1]Y_c \\
2 & \quad [1]Y_f \\
1 & \quad [1]Y_d \\
1 & \quad [1]Y_e \\
1 & \quad [1]Y_b \\
1 & \quad [1]Y_c \\
1 & \quad [1]Y_f \\
\end{align*} \]

The order in which the labels are assigned is not important here, provided columns and rows follow the same order. However, for consistency with sections let us assign the node numbers in the directions of the branch currents of Fig. 7.5, which also shows the numerical values of the admittances. Combining those elements of the above matrices having identical row and column labels gives:

\[ \begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & \quad (Y_e + Y_d + Y_f) & -Y_d & -Y_e & -Y_f \\
2 & -Y_d & \quad (Y_b + Y_d + Y_e) & -Y_b & -Y_e \\
3 & -Y_e & -Y_b & \quad (Y_a + Y_b + Y_c) & 0 \\
4 & -Y_f & -Y_e & 0 & \quad (Y_e + Y_f + Y_c) \\
\end{bmatrix} \]
Modification is easy:

ex: Remove Line 2-3: \((-j4)\)

\[
\begin{bmatrix}
-14.5 & 8 & 4 & 2.5 \\
8 & -13 & 0 & 5 \\
4 & 0 & -4.8 & 0 \\
2.5 & -5 & 0 & -8.3 \\
\end{bmatrix}
\]

\([Y_{bus}]\) is SPARSE in general.

Typical "grid" system being analyzed
will have 100's or 1000's of buses.

For \(y_{12} = 0\), no line or xfar from 1-2.

\[\begin{align*}
y_{12} &= 0 \\
y_{21} &= 0
\end{align*}\]
Typically, only 2-5 buses are connected to a given bus, so most off-diagonal entries of $[Y]$ are 0.

When many entries of a matrix are 0, it's a sparse matrix.

- Don't have to store zero values.
  - Single-precision complex values: 8 bytes.

For 10,000 bus system:

$\Rightarrow$ 800 MB of RAM.

Use linked-list storage, only store the non-zero values.
If each bus is connected to 4 others, 5
then each row has 5 entries.

only 400 KB needed.

\[ E = [Y]^T \] is a full matrix.

Impact:

\[ B = [Y] \]

Must use factorization methods to obtain desired entries.

Can find \( z \).
Kron Reduction - System Reduction
- Kron Elimination


Possible to reduce to equiv system of fewer nodes.
Goal: Only buses of interest need be observable.

Constraint: Must retain source nodes (nodes at which current is being injected).

Steps:
1) Reorder system - move buses to keep top, i.e. 1...K
   Remaining L...Z nodes are absorbed into system.
2) Perform Kron Reduction.
\[
\begin{bmatrix}
[L] & [M]
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix} =
\begin{bmatrix}
I_A \\
I_x
\end{bmatrix}
\]

\[
Y_{bus}
\]

\[
I_A = K(V_A + LVB)
\]

\[
I_x = LTV_A + MV_B
\]

Since \( I_x = \begin{bmatrix} 0 & \vdots \end{bmatrix} \)

\[
\begin{align*}
\text{1.} & \\
\text{2.} &
\end{align*}
\]
(3) \(-L^TVA = MV^B \leftarrow \text{From Egn. (2)}\) for \(I_x = 0\).

(4) \(-M^LTV^A = V^B \leftarrow \text{premultiply both sides by } M^{-1}\).

Substituting \(V^B\) into Egn. (1),

\[ I_A = KV^A - LM^LTV^A \]

\[
[I_A] = [K - LM^L][V^A]
\]

The \([Y\text{bus}]\) for this reduced system is thus implied to be \([K - LM^L]\).

Derivation assumes bilateral system (note \(L, L^T\))
Reduced $[\text{Y}_{\text{bus}}]$ is

$[\text{Y}_{\text{bus, reduced}}] = K - LM^{-1}LT$

**IMPORTANT OBSERVATION:**
If $L$ & $LT$ are off-diagonals, then this eqn. only valid for bilateral system!