Topics for Today:

- Announcements
  - Partnered homework now possible, now thru end of semester.
  - Nov 2\textsuperscript{nd} - Detailed Term Project outline (in format of report \textit{Table of Contents}) + complete list of references.
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - ASPEN software - arranging to run off of MTU server via internet.
  - Office: EERC 614. Phone: 906.487.2857
  - Recommended problems & all solutions: Ch.7, 8 solns now posted.

- Chapter 7 - Network Equations, Admittance Approaches
  - Kron Reduction
  - Mutual Inductance
  - Solution of matrix equations (system of linear equations)
    - Pre-multiply by inverse
    - in-situ methods
      - Gauss Elimination, Gauss-Jordan Elimination
      - LU Factorization (method used by Matlab \texttt{\backslash} function
  - Basic fault calculation - balanced 3-phase "bolted" fault
  - Upcoming homework - intro to Matlab, matrices, plotting.
A term project shall be done in lieu of a final exam. The project you choose:
- must be of topical interest,
- must relate to course material of EE5200.
- must demonstrate a graduate student level of mastery and application of the related
  concepts and theories.
- is sufficiently researched (referenced) and documented, including at least one in-depth
  analysis and presentation of the concepts of the journal paper that is most related to this
  work.
- length of body of report: approximately 10 pages of text (not including figures, tables,
  or equations).

Time line and required submissions are as follows, all deliverables contribute to the grade
of your term project, i.e. 15% of your course grade. Approximate schedule is:
- Week 6 (Friday): submit short e-mail with idea(s) requesting instructor feedback.
- Week 7 (Friday): submit formal outline of project and list of key references.
- Week 9: submit updated outline of project and complete reference list.
- Week 11: journal paper analysis and presentation (also counts as mini-lecture)
- Week 12: Submit rough draft of project report
- Week 14: Submit final report/deliverable.
- Finals week: present/demonstrate project during final exam time-slot.

Report Outline:
Front Matter:
- Title Page
- Executive Summary (not needed for initial draft)
- Table of Contents (use as "working outline")
Body of report:
- Introduction (brief overview of project: problem area, motivation, overview of project)
- Background
  - literature search, most important references
  - Presentation of key concepts connected with project
  - Identification of existing voids or weaknesses, and resulting opportunity
- Proposed Approach
  - Overview of basic idea that you will develop and implement
  - Development and implementation details
- Implementation (may not be complete in draft versions)
- Results (Expected Results in draft versions)
- Conclusion
- Recommendations for Continued Work

Supplemental Information:
- Reference List (number references [1], [2], etc, in order of first author's last name)
- Appendices as required to document details

Suggested layout:
- Font: 11-pt CG Times w/1.25-1.5 line spacing; or 10-pt comic or ariel w/1.0-1.25 line space
- Page layout: 1" margins, include page numbering within margin area.
Kron Reduction - System Reduction
- Kron Elimination

Possible to reduce to "equiv system" of fewer nodes.
Goal: Only buses of interest need be observable.

Constraint: Must retain source nodes (nodes at which current is being injected).

Steps:

1) Reorder system—move buses to kept to top, i.e. 1,...,k
   Remaining L...Z nodes are absorbed into system.

2) Perform Kron Reduction.
\[
\begin{bmatrix}
[K] & [L] & [V_A] \\
\end{bmatrix}
\begin{bmatrix}
Y_{Bus} \\
V \\
\end{bmatrix} =
\begin{bmatrix}
I_A \\
I_x \\
\end{bmatrix}
\]

\( I_A = KV_A + LV_B \)

\( I_x = L^TVA + MVB \)

Since \( I_x = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \)
(3) \(-L^TVA = MV_B \quad \text{From Eqn. 2 for } I_X = 0.\)

(4) \(-M'LTVA = V_B \quad \text{ premultiply both sides by } M^{-1}\)

Substituting \(V_B\) into Eqn. (1),

\[
I_A = KVA - LM'LTVA
\]

\[
[I_A] = [K-LM'LT][V_A]
\]

The \([Y_{bus}]\) for this reduced system is thus implied to be \([K-LM'LT]\).

Derivation assumes bilateral system (note \(L, LT\)).
Reduced \([Y_{bus}]\) is

\[
\begin{bmatrix}
Y_{bus} \\
Y_{Reduced}
\end{bmatrix} = K - LM^{-1}LT
\]

**Important Observation:**

If \(L\) and \(LT\) are off-diagonals, then this eqn. only valid for bilateral System!
inductive coupling!
Mutual Inductance

\[ \begin{align*}
I_1 & \rightarrow V_1 \\
I_2 & \rightarrow V_2
\end{align*} \]
**MUTUAL INDUCTANCE**

- See also handout on Basic Magnetic Circuits

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**Fundamental definition of inductance:**

\[
L = \frac{\tau}{i} = \frac{N\Phi}{i}
\]

- **Self-Inductance**

\[
L_{11} = \frac{N_1 \Phi_{11}}{i_1} = \frac{\tau_{11}}{i_1} = \frac{N_1^2}{R}
\]

- **Mutual Inductance**

\[
L_{12} = \frac{N_1 \Phi_{12}}{i_2} = \frac{\tau_{12}}{i_2} = \frac{N_1 N_2}{R}
\]

\[
L_{21} = \frac{N_2 \Phi_{21}}{i_1} = \frac{\tau_{21}}{i_1} = \frac{N_2 N_1}{R}
\]

\[
L_{22} = \frac{N_2 \Phi_{22}}{i_2} = \frac{\tau_{22}}{i_2} = \frac{N_2^2}{R}
\]
How to Use the Concept of Mutual Inductance

Two-Port Device:

\[
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\]

Note: Reference direction of currents is into terminals at (+) side of voltage.

In time domain:

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix} \begin{bmatrix}
\frac{di_1}{dt} \\
\frac{di_2}{dt}
\end{bmatrix}
\]

In phasor domain:

\[
\begin{bmatrix}
\tilde{V}_1 \\
\tilde{V}_2
\end{bmatrix} = \begin{bmatrix}
jωL_{11} & jωL_{12} \\
jωL_{21} & jωL_{22}
\end{bmatrix} \begin{bmatrix}
\tilde{I}_1 \\
\tilde{I}_2
\end{bmatrix}
\]

Also of note: In some texts, since \( L_{12} \) and \( L_{21} \) are mutual inductances, they are called \( M_{12} \) and \( M_{21} \). Same thing.
$[Z'] = [Y]$

$$\begin{bmatrix} y_1 & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$$

\[ \xi 7.2 \]

of text.
Assume high \( Y/R \) (\( R \to 0 \))

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
Z
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} \Rightarrow \begin{bmatrix}
Y \\
Y
\end{bmatrix} \begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

pre-multiply both sides by \( [Z]^T \)

$$[y^T] = [3]$$

If $[3]$ is symmetric about the main diagonal (bilateral), then use either row or col.

$$\begin{array}{cccc}
Z_{11} & Z_{12} & \cdots & Z_{1n} \\
Z_{21} & Z_{22} & \cdots & Z_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{n1} & Z_{n2} & \cdots & Z_{nn}
\end{array}$$
$z_{nn} = z_{TH}$ at bus $n$.

- Off-diagonal $z$'s represent the mutual impedances between bus $n$ and all other buses.

$\tilde{V}_F = \text{Voltage at bus (Voc) pre-fault}$

$\bar{I}_{sc} = \frac{\tilde{V}_F}{z_{nn}}$

$\bar{I}_{ij} = -I_{sc}$

Grid System
"Prefault" Situation

\[ [Z] = [Y]^{-1} \]

Lines, XFMRs, LOADS, SHUNT CAP/REACTORS

(for this case, also the Gen Impedances) *

Ex: Fig 7.5

IF there is a fault at bus n in system,

\[ \tilde{V}_F = \tilde{I}_F \left( \frac{Z}{Z_{mn}} \right) \]

Often assume that \( V_F = 1.05 \) p.u.
What happens at other buses during the fault? All bus voltages will dip. How much?

\[ V_F \]

During Fault

\[ E_L = V_F - \frac{I_F}{Z_{in}} \]

\[ = V_F - \frac{V_F}{Z_{nn}} \cdot Z_{in} = V_F - \frac{Z_{in}}{Z_{nn}} V_F \]

\[ \vec{I}_F = \frac{\vec{V}_F}{Z_{nn}} \]

"RAKE EQUIV"
\[ \begin{bmatrix} \ldots \ldots \ Z_{in} \\ Z_{inn} \\ \vdots \\ Z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \text{In, } Z_{in} \\ \text{In, } Z_{inn} \\ \text{In, } Z_{nn} \end{bmatrix} \]

\[ V_{i} = V_{F} - I_{F} Z_{in} \]

\( V_{drops \ due \ to \ -I_{F}} \)

\( \text{In} = -I_{F}, \text{ injected into bus n} \)
Fault Contributions (i.e. current)

Must Know

\[ I_{\text{Fault contr}} \]: Are CBS going to be able to interrupt?

- Relay engineers must know all current flows.
Refering to Ybus, current contribs are

\[ [z]^{-1} = \begin{bmatrix} y_{mm} \\ \vdots \end{bmatrix} \]

\[ y_{ng} \quad \text{only non-zero values in row n.} \]

\[ I_{From} = \begin{cases} (V_g - V_n) (-y_{ng}) \\ (V_j - V_n) (-y_{nj}) \\ (V_k - V_n) (-y_{nk}) \end{cases} \]
P.6 method ok for Short-Line Connections.

- What about Pi-eqvin Line
- Shunt Load
- Shunt Cap/React?

*must include effect of Shunt Cap, unless $V_n = 0$.

More on this later, and in EE5240.

Not a contributor to 60-Hz Short-Circuit Current
Begin with practical use of $[Z]$

*Thevenin Impedance: Main diagonal element of $[Z_{bus}]$*

Useful to know $Z_{TH}$ at bus

Prefault Voltage $\bar{V}_{TH}$

Fault Current $\bar{I}_F = \frac{\bar{V}_{TH}}{Z_{KK}}$