Topics for Today:

- Announcements
  - Matlab - last reminder to purchase (online students).
  - Office hrs: 2:00-3:00pm M,W,F
  - Office: EERC 614. Phone: 906.487.2857
  - Recommended problems from Ch.3, solutions posted

- Transformers and circuits w/ transformers
  - Paralleling of transformers
    - Proportioning of MVA flow for unequal MVA size, unlike impedances
  - Circuit calculations for above cases
  - Design and operations issues
  - Phase shifting transformers
  - Remaining topics will be covered in context of system operation & analysis, i.e. Chapters 7 and 8.
    - Per phase Pi-equivalent for off-nominal turns ratio, phase shifts, etc.
    - Incorporation in system admittance matrix for short-circuit and load flow
Next Lecture: Synchronous Machines - Chapter 3
- Recommended problems & solns for Ch.3 are posted.
- Phasor diagrams - unity, lag, lead
- Salient rotor machines - calculation with Xd and Xq.
- Calculation Example(s)
- P & Q flows thru transmission lines
- More on admittance matrix [Y] construction

"Intro" to Synch Machines - L12
- Dr. Bohmann
- Physical structure
- A-BC-Rotor \( \Rightarrow \) d-q per phase analysis.
CASE 1

Paralleling

- "Unlike" impedances.
- Equal turns ratios which match V BASE of system.

KEY: $Z_{sc}$ of $T_1$ and $T_2$ equal on base of respective xfrms
CASE 2

- Turns ratio (voltage ratio) of xfmr is not equal to ratio of VBASE of system.

\[ V_B = 115 \text{ KV}_{ll} \quad \text{and} \quad V_B = 69 \text{ KV}_{ll} \]
Parallelizing Transformers of Unlike Turns Ratio

\[ \frac{N_1}{N_2} = \frac{V_1}{V_2} \]

What happens for \( \frac{N_1'}{N_2} \neq \frac{N_1}{N_2} \)?

Replace \( T_2 \) with 2 XFMRS: First is same ratio as \( T_1 \), \( \frac{N_1}{N_2} = \frac{N_1'}{X} \)

Second XFM has ratio of off-nominal turns

For unit equivalent:

\[ \frac{1}{a} = \frac{N_2 N_1'}{N_1 N_2'} = \frac{N_2 N_1'}{N_1 N_2} \]

\[ a = \frac{N_1 N_2'}{N_2 N_1'} = \text{p.u. turns ratio} \]

Three Methods to Analyze:

1) Admittance Method
2) Circuit Theory
3) Circulating Current Method (Approximate)
So, we must find a way to model $Y$ for $Y_{12}$ as $Y_a$ varies.

\[ I_1 = Y_{11}V_1 + Y_{12}V_2 \]
\[ I_2 = Y_{12}V_1 + Y_{22}V_2 \]
\[ S_1 = -S_2 \]
\[ S_1 = \frac{V_2}{a}I_1^* \quad \quad \quad S_2 = V_2I_2^* \]

\[ \frac{V_2}{a}I_1^* = -V_2I_2^* \]
\[ I_1^* = -aI_2^* \]
\[ I_1 = -a^*I_2 \]

\[ I_1 = (V_1 -\frac{V_2}{a})Y = Y_{11}V_1 - \frac{Y_{12}}{a}V_2 = -a^*I_2 \]

\[ I_2 = L\frac{-I_1}{a^*} = -\frac{Y_{11}}{a^*}V_1 + \frac{Y_{12}}{aa^*}V_2 \]

\[ Y_{11} = Y \quad Y_{12} = -\frac{Y}{a} \]
\[ Y_{21} = -\frac{Y}{a^*} \quad Y_{22} = \frac{Y}{aa^*} = \frac{Y}{|a|^2} \]
2) **Circuit Theory Approach**

\[ V_1 = 1 \angle 0^\circ \]

\[ V_2 = \frac{(8 + j5)(1 \angle 0^\circ)}{1.8 + j5} = 0.94 - j0.08 = 0.9434 \angle 48.8^\circ \]

\[ I_2 = \frac{0.94 - j0.08}{1.8 + j5} = 1 \angle 36.87^\circ \]

\[ S_0 = VI^* = 0.9434 \angle 48.8^\circ (1 \angle 36.87^\circ) = 0.9434 \angle 32^\circ = 0.88 + j0.5 \]

\[ S_1 = 0.4 + j0.25 \]

\[ S_2 = 0.4 + j0.25 \]

\[ S_{\text{TOTAL}} = \frac{S_1 + S_2}{2} \]

\[ I_1 = \frac{1}{1.8 + j5} = 1(0.8 - j0.6) = 0.8 - j0.6 \]

\[ S_{1a} = \frac{V_1 I_1^*}{2} = 4 + j3 \]

\[ S_{1b} = \frac{V_1 I_1^*}{2} = 0.4 + j1.3 \]

*Difference due to Xfmr Inductance*
Now add tap changer \( a = 1.05 \)

\[
I_1 = I_{la} + I_{lb} \\
I_1 = \frac{1-V_2}{j \cdot 2} + \frac{1 - \frac{V_2}{1.05}}{j \cdot 2} \\
I_2 = \frac{1-V_2}{j \cdot 2} + \frac{1 - \frac{V_2}{1.05}}{j \cdot 2} \left( \frac{1}{1.05} \right) \\
V_2 = I_2 (0.8 + j \cdot 5) = \left[ \frac{1-V_2}{j \cdot 2} + \frac{1 - \frac{V_2}{1.05}}{j \cdot 2} \left( \frac{1}{1.05} \right) \right] (0.8 + j \cdot 5) \\
V_2 = \left( 1-V_2 + 1 - \frac{V_2}{1.05} \right) (0.8 + j \cdot 5) = (1.952 - 1.907V_2) (4.717 - 58^\circ) \\
V_2 = -1.907V_2 (4.717 / -58^\circ) + (1.952) (4.717) / -58^\circ \\
V_2 = -8.995 / -58^\circ V_2 + 9.207 \cdot -58^\circ \\
V_2 = \frac{9.207 / -58^\circ}{1 + 8.995 / -58^\circ} = \frac{9.207 \cdot -58^\circ}{9.563 / -52.9^\circ} = 0.963 \cdot -5.1^\circ \\
I_{la} = \frac{1-V_2}{j \cdot 2} = 0.473 / -25.54^\circ \\
I_{lb} = \frac{1 - \frac{V_2}{1.05}}{j \cdot 2} = 0.59 \cdot 46.75^\circ \\
S_{la} = V_1 I_{la}^* = 0.427 + j \cdot 204 \\
S_{lb} = V_1 I_{lb}^* = 0.4067 + j \cdot 4324
\[ \tilde{S}_1 = V_2 I_1^* = (0.963 \angle -50^\circ) \times 0.4732 \angle 25.5^\circ \]
\[ = 1.427 + j0.16 \]

\[ \tilde{S}_2 = \frac{V_2 I_2^*}{\alpha} = 0.963 \angle -51^\circ \left( \frac{0.59 \angle 46.75^\circ}{1.05} \right)^* = 0.406 + j0.3616 \]

\[ \tilde{S} = V_2 I_2^* = 0.963 \angle 51^\circ \left( \frac{V_2^*}{2} \right)^* = 0.983 \angle 32^\circ = \tilde{S}_1 + \tilde{S}_2 \]

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**Before TC**

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<thead>
<tr>
<th></th>
<th>V_1</th>
<th>1</th>
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<tbody>
<tr>
<td>V_2</td>
<td>0.943 \angle 4.86^\circ</td>
<td>0.963 \angle 4.81^\circ</td>
</tr>
<tr>
<td>P_1</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>R_2</td>
<td>0.4</td>
<td>0.418</td>
</tr>
<tr>
<td>Q_1</td>
<td>0.75 \times V_2</td>
<td>0.135</td>
</tr>
<tr>
<td>Q_2</td>
<td>0.125</td>
<td>0.375</td>
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**After TC**

<table>
<thead>
<tr>
<th>Circ. Approx.</th>
<th>Ckt. Analysis</th>
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<tr>
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\[ 0.398 \times 4.27 \times V_2 \]

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9.27 Tap Changing XFRM shifted to XFRM with higher tap setting.

So:

- Q is shifted to XFRM of higher tap setting.
- P is still divided almost evenly.
Approximate solution:

Superposition of circulating current

\[ I_{\text{circ}} = \frac{0.05 \angle 0^\circ}{j \cdot 4} = 0.125 \angle -90^\circ = -j \cdot 0.125 \]

First, look at XFMR before load applied.

From first example,

\[ I_{1a} = 0.4 - j \cdot 0.3 + j \cdot 1.25 = 0.4 - j \cdot 0.75 \]

\[ I_{1b} = 0.4 - j \cdot 0.3 - j \cdot 1.25 = 0.4 - j \cdot 0.625 \]

By superposition,

\[ V_2 = V_2 \text{ before tap change} + \frac{\Delta V(j)X_t}{jX_t + jX_2} \frac{Z_{\text{LINE}}}{Z_{\text{TOTAL}}} \]

\[ \Delta V = \frac{\Delta V(jX_t)}{jX_t + jX_2} \]

\[ V_2 = 0.943 \angle 48^\circ + \frac{0.05(j)}{j \cdot 4} \frac{0.8 + j \cdot 0.5}{0.8 + j \cdot 0.6} \]

\[ V_2 = 0.963 \angle 48.7^\circ \]
$S_{1a} = V_2 I_{1a} = (0.963 \angle 48.7^\circ)(437 \angle 23.63^\circ) = 421 \angle 18.76^\circ$

$S_{2a} = V_2 J_{2a} = (0.963 \angle 48.7^\circ)(1.584 \angle 46.73^\circ) = 562 \angle 41.86^\circ$

**Phase Shifting XFMK**

\[ a = 1 / \sqrt{3} = e^{-j \pi / 6} \]

Without phase shifter,

\[ P_{load} = \frac{V_2^*}{\alpha^*} = 1 (0.8 + j 0.6) = 0.8 + j 0.6 \]

\[ S_a = 0.9 + j 3 \]
\[ S_b = 0.4 + j 3 \]
\[ I_a = 0.4 - j 3 \]
\[ I_b = 0.4 - j 3 \]

With phase shifter,

\[ \frac{V_1 - I_j}{\sqrt{3}} + \frac{V_1 - \frac{1}{\sqrt{3}}}{j} (\frac{1}{\sqrt{3}}) = \frac{1}{0.8 + j 0.6} = I_2 \]

\[ V_1 (1 + \frac{1}{\sqrt{3}}) = 1 - \left(\frac{1}{\sqrt{3}}\right)^2 = j \sqrt{8 + 16} \]

\[ V_1 (1.999 \angle 1.5^\circ) = 1 - (1.999 \angle 1.5^\circ) = 1.5313 + 1.9945 \]

\[ V_1 = 1.031 \angle 173^\circ = 1.031 + j 0.13 \approx \]

\[ I_2 = 1.028 + j 0.6 \]

\[ j \cdot 1 = 1.922 \angle 60.32^\circ = 1.59 \angle 60.32^\circ \]
\[ I_a = \frac{1.031 + j0.013}{j1} = 1.131 - j0.311 \]

\[ I_b/a^* = I_2 - I_a = 0.8 - j0.6 - 1.131 + j0.311 = 0.669 - j0.289 \]

\[ S_a = 1.131 + j0.311 \]
\[ S_b = 0.669 + j0.289 \]

\[ \text{NOTE: SHIFT IN POWER FLOW THRU XFMRS} \]

<table>
<thead>
<tr>
<th>BEFORE</th>
<th>AFTER</th>
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<tbody>
<tr>
<td>( I_a )</td>
<td>( 1.131 - j0.311 )</td>
</tr>
<tr>
<td>( I_b/a^* )</td>
<td>( 0.669 - j0.289 )</td>
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<tr>
<td>( P_a )</td>
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<tr>
<td>( P_b )</td>
<td>( 0.669 )</td>
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<tr>
<td>( Q_a )</td>
<td>( 0.311 )</td>
</tr>
<tr>
<td>( Q_b )</td>
<td>( 0.289 )</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>( 1.031 \angle 2.2^\circ )</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>( 1.0 \angle 6^\circ )</td>
</tr>
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if \( a \) is at positive angle, power flow through phase shifting XFMR increases
PS XFMR -
Similar to this
Simple concept
(Short T-Line).

Pflow
increases
with \(s_1-s_2\).

\[ P_{12} = \frac{V_1 V_2}{x} \sin(s_2-s_1) \]
Phase Shift Transformers (Fig. 2.22)

Read §2.9!
Circulating Vars

Energization Testing of Var Meters
\[ V_1 + \frac{V_1}{C} \rightarrow 5_1 = 5_2 \rightarrow \]

\[ V_1 I_1^* = V_2 I_2^* \]

\[ I_2^* = \frac{V_2}{V_1} = c \]

\[ \frac{I_2^*}{I_1^*} = \overline{c} \]