Topics for Today:

- **Announcements**
  - Detailed term project outlines (i.e. Table of Contents + List of references)
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - ASPEN software - arranging to run off of MTU server via internet.
  - Office: EERC 614. Phone: 906.487.2857
  - Recommended problems & all solutions: Ch.7 solns posted.

- **Chapter 7 - Network Equations, Admittance Approaches**
  - Overview of off-nominal xfmrs
  - Double-circuit lines - mutual coupling
  - Network Reduction (Kron Reduction)
  - Solution of matrix equations (system of linear equations)
  - Lead-in to Short-circuit and other formulations.
  - Upcoming homework - intro to Matlab, matrices, equations.
Basis Approach: Develop Π-Equiv and handle just like T-Line.

One-Line:

\[ \text{l} \xrightarrow{1} \text{3} \xrightarrow{2} \]

per-unit per-phase

Top-CHANGERS
- LTC's
- Phase-Shift

Nominal Turns Ratio \[ \pm \text{Adjustment in phase angle (PS)} \]

{}
Tap Changing XFMRs - Variations (p.u. representations)

\[ y_{sc} = \frac{1}{R + jX} \]

1. \( c \) is off-nominal turns ratio. In general, \( c \) is complex.
2. \( c \) is real for LTC.
3. \( c \) is complex for PS.
4. If \( |c| \neq 1 \) then magnitude change.
   If \( c \) is complex, phase shift.

\[ (R + jX) \]

\[ \frac{1}{c} \]

\[ \frac{y_{sc}}{C} \]
TAP-CHANGERS

On One-Line Diags:

Conceptually:

In per unit, nominal transformation "disappears"
Generically, we can describe this box as a 2-node $[Y]$

where

$$
\begin{bmatrix}
  y_{11} & y_{12} \\
  y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
  \overline{V}_1 \\
  \overline{V}_2
\end{bmatrix}
= 
\begin{bmatrix}
  \overline{I}_1 \\
  -\overline{I}_2
\end{bmatrix}
$$
Standard Approach:

\[
\begin{bmatrix}
Y_11 & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
= \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

Goal:

\[
y_{11} = Y_{SER} + y_{SH1}
\]
\[
y_{12} = -Y_{SER}
\]
\[
y_{21} = -Y_{SER}
\]
\[
y_{22} = Y_{SER} + y_{SH2}
\]

Injected currents!
Strategically using shorts, we can isolate on the values of \([Y]\).

\[
y_{11} = \frac{-I_1}{V_1}, \quad V_2 = 0
\]

\[
y_{22} = -\frac{I_2}{V_2}, \quad V_1 = 0
\]

\[
Y_{\text{EQ}} = \frac{1}{Z_{\text{EQ}}/1c_l^2} = c_l^2Y_{\text{EQ}}
\]

\[
\frac{1}{Z_{\text{EQ}} + jX_{\text{EQ}}} = \frac{-I_1 - c_lV_2 + c_lI_2}{Z_{\text{EQ}}}
\]
\[ I_1 = -\frac{C V_s}{2\varepsilon_0}, \quad I_2 = -\frac{C^*}{2\varepsilon_0} \]

\[ \frac{I_{1*}}{I_{2*}} = \frac{c^*}{C^*} = \frac{-\frac{C V_s}{2\varepsilon_0}}{-\frac{C^*}{2\varepsilon_0}} = \frac{1C^*}{2\varepsilon_0} V_s \]

Note: \( I_{1*} = c^* \)
\[ y_{12} = \left. \frac{\tilde{I}_1}{\tilde{V}_2} \right|_{\tilde{V}_1 = 0} = \frac{-c \tilde{V}_2 / Z_{EQ}}{\tilde{V}_2} = -c Y_{EQ} \]

\[ y_{21} = \left. \frac{-\tilde{I}_2}{\tilde{V}_1} \right|_{\tilde{V}_2 = 0} = \frac{-c \tilde{I}_1}{\tilde{V}_1} = -c Y_{EQ} \]

Note: Ideal XFRH, by definition, has "C" is voltage ratio.

\[ C = \frac{\tilde{V}_1}{\tilde{V}_2} \]

\[ C = \frac{\tilde{I}_2}{\tilde{I}_1} \Rightarrow \frac{\tilde{I}_2}{\tilde{I}_1} = C \]

\[ S_{in} = \tilde{V}_1 \tilde{I}_1^* = \tilde{V}_2 \tilde{I}_2^* = S_{out} \]
If we "reverse engineer" our \([Y]\) into an equivalent 2-bus network, then

\[ \begin{align*}
  \vec{I}_1 &\rightarrow C Y_{EQ} \rightarrow \vec{I}_2 \\
  \vec{V}_1 &\rightarrow Y_{EQ}(1-C) \rightarrow \vec{V}_2 \\
  \end{align*} \]
Observations:

- LTC (RTU) has a c that is Real.
- Transfer Admittances
  \[ C_{y\alpha} = C_{x\gamma} Y_{\alpha\gamma} \]
  has complex c.
  \[ y_{12} = y_{21} \]

- Phase Shifter (PS) has complex admittances
  \[ C_{y\alpha} \neq C_{x\gamma} \]
  \[ y_{12} \neq y_{21} \]

\[ \{Y\} \text{ not symm. (about main diag.)} \]

Not Bilateral.
Transformer LTC's in the CDF File Format

Tap and impedance location specified in first two entries in branch data section.
- entry 1 is bus non-unity tap is connected to
- entry 2 is bus device impedance is connected to

```
       C:1
      /   \
     /     \ R+|X
    /       \  
   /         Z
  /           \ 
 /             \ 
From

```

Complex turns ratio due to phase shifting transformer split to two entries
- entry 15 is transformer final turns ratio
- entry 16 is transformer (phase shifter) final angle

Examples:

<table>
<thead>
<tr>
<th>4</th>
<th>0.975:1</th>
<th>j0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry:</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3</th>
<th>j130:1</th>
<th>j0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry:</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>
Mutual Inductance

- See also handout on Basic Magnetic Circuits

\[ L = \frac{\overline{I}}{i} = \frac{N\Phi}{i} \]

- Fundamental definition of inductance:

\[ L_{11} = \frac{N_1 \Phi_{11}}{i_1} = \frac{\overline{I}_{11}}{i_1} = \frac{N_1^2}{R} \]

Self-Inductance

\[ L_{21} = \frac{N_2 \Phi_{21}}{i_1} = \frac{\overline{I}_{21}}{i_1} = \frac{N_2 N_1}{R} \]

Mutual Inductance

\[ L_{12} = \frac{N_1 \Phi_{12}}{i_2} = \frac{\overline{I}_{12}}{i_2} = \frac{N_1 N_2}{R} \]

\[ L_{22} = \frac{N_2 \Phi_{22}}{i_2} = \frac{\overline{I}_{22}}{i_2} = \frac{N_2^2}{R} \]

Self-Inductance
How to Use the Concept of Mutual Inductance

Two-Port Device:

\[
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\]

Note: Reference direction of currents is into terminals at (+) side of voltage.

In time domain:

\[
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\begin{bmatrix}
di_1/\text{dt} \\
di_2/\text{dt}
\end{bmatrix}
\]

In phasor domain:

\[
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
jωL_{11} & jωL_{12} \\
jωL_{21} & jωL_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

Also of note:

In some texts, since \(L_{12}\) and \(L_{21}\) are mutual inductances, they are called \(M_{12}\) and \(M_{21}\). Same thing.
\[ \mathbf{x}' = \mathbf{Y} \mathbf{Y}^T \]

\[
\begin{bmatrix}
 y_1 \\
y_2 \\
y_3 \\
y_4
\end{bmatrix}
\begin{bmatrix}
 v_1 \\
v_2
\end{bmatrix}
= 
\begin{bmatrix}
 i_1 \\
i_2
\end{bmatrix}
\]

\[ \text{§7.2 of text.} \]
Assume high \( \frac{V}{R} \) (\( R \rightarrow 0 \))

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} = 
\begin{bmatrix}
Z
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} = 
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

pre-multiply both sides by \( \begin{bmatrix} Z \end{bmatrix} \)
Possible to reduce to "equiv system" of fewer nodes.
Goal: Only bases of interest need be observable.

Constraint: Must retain source nodes (nodes at which current is being injected).

Steps:
1) Reorder system to move buses to top, i.e. 1,...,K
2) Perform Krzan Reduction

Remaining L - K nodes are absorbed into system.

Bethlehem: Instructors. Bruce Mark. Phone: (506) 467-3357 Email: bmark@csun.edu
\[
\begin{bmatrix}
[K] & [L] \\
[L^T] & [M]
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix}
= 
\begin{bmatrix}
I_A \\
I_x
\end{bmatrix}
\]

\[I_A = KV_A + LV_B\]

\[I_x = L^TV_A + MV_B\]

Since \(I_x = \begin{bmatrix} 0 \\ \ldots \\ 0 \end{bmatrix}\)
\[ 3 \] \quad -L^T V_a = M V_B \quad \text{From Eqn. (2)}
\text{for } I_x = 0.

\[ 4 \] \quad -M^L L^T V_a = V_B \quad \text{ premultiply both sides by } M^{-1}.

Substituting \( V_B \) into Eqn. (1),

\[
I_a = K V_a - L M^L L^T V_a
\]

\[
[I_a] = [K - L M^L L^T][V_a]
\]

The \([Y_{bus}]\) for this reduced system is thus implied to be \([K - L M^L L^T]\).

Derivation assumes bilateral system (note \( L, L^T \)).
Reduced \([\text{Ybus}]\) is

\[
[\text{Ybus Reduced}] = K - \text{LM}^{-1}\text{L}^T
\]

**Important Observation:**
If \(L\) & \(L^T\) are off-diagonals, then this eqn. only valid for bilateral system.