\[ \tilde{S}_1 = V_2 I_{1*} = (0.963 \angle -50^\circ ) \times 4732 \angle 25.8^\circ = 1427 + j.16 \]

\[ \tilde{S}_2 = \frac{V_2 I_{1*}}{2} = 0.963 \angle -51^\circ \left( \frac{59 \angle -46.75^\circ}{1.05} \right)^* = 0.406 + j.3616 \]

\[ \tilde{S} = V_2 I_{2*} = (0.963 \angle 51^\circ ) (\frac{V_2}{2})^* = 0.983 \angle 32^\circ = \tilde{S}_1 + \tilde{S}_2 \]

<table>
<thead>
<tr>
<th>Before TC</th>
<th>After TC: Circ. Approx.</th>
<th>Ckt. Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>0.943 \angle -4.86^\circ</td>
<td>0.963 \angle -51^\circ</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>0.4</td>
<td>0.398</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>0.4 ( V_{at} )</td>
<td>0.418</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>0.25 ( V_2 )</td>
<td>0.135</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>0.25</td>
<td>0.375</td>
</tr>
</tbody>
</table>

9.2 Tap Changing XFMR Shifted Q to XFMR with top run to higher rated secondary voltage higher tap setting:

So:
- Q is shifted to XFMR of higher tap setting.
- P is still divided almost evenly.
Now add tap changer \( a = 1.05 \)

\[
I_1 = I_{1a} + I_{1b}
\]

\[
I_1 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}
\]

\[
I_2 = \frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}\left(\frac{1}{1.05}\right)
\]

\[
V_2 = I_2 (1.8 + j.5) = \left[\frac{1 - V_2}{j.2} + \frac{1 - \frac{V_2}{1.05}}{j.2}\left(\frac{1}{1.05}\right)\right] (1.8 + j.5)
\]

\[
V_2 = \left(1 - V_2 + \frac{V_2}{1.05}\right) (1.8 + j.5) = (1.952 - 1.907V_2) (1.8 + j.5)
\]

\[
v_2 = -1.907V_2 (4.717 \angle -58^\circ) + (1.952V_2 (4.717 \angle -58^\circ)
\]

\[
v_2 = 9.207 \angle -58^\circ
\]

\[
V_2 = \frac{9.207 \angle -58^\circ}{1 \angle -58^\circ} = 9.207 \angle -58^\circ
\]

\[
I_{1a} = \frac{1 - V_2}{j.2} = 0.476 \angle 25.54^\circ
\]

\[
I_{1b} = \frac{1 - \frac{V_2}{1.05}}{j.2} = 0.59 \angle 46.75^\circ
\]

\[
S_{1a} = V_1 I_{1a}^* = 0.427 + j.204
\]

\[
S_{1b} = V_1 I_{1b}^* = 0.4067 + j.4324
\]
2) **Circuit Theory Approach**

\[ V_1 = 1 \angle 0^\circ \]

\[ V_2 = \frac{(8 + j5)(1 \angle 0^\circ)}{1.8 + j.6} = 0.94 - j.08 = 0.9434 \angle -4.86^\circ \]

\[ I_2 = \frac{0.94 - j.08}{1.8 + j.5} = 1 \angle -36.87^\circ \]

\[ S_0 = V_1 I_2^* = 0.9434 \angle -4.86^\circ (1 \angle 36.87^\circ) = 0.9434 \angle 32^\circ = 0.8 + j.5 \]

\[ S_1 = 0.8 + j.25 \]

\[ S_2 = 0.8 + j.25 \]

\[ \frac{S_1 + S_2}{2} = \frac{S_{\text{Total}}}{2} \]

\[ I_1 = \frac{1}{0.8 + j.6} = 1 (0.8 - j.6) = 0.8 - j.6 \]

\[ S_{1a} = \frac{V_1 I_1^*}{2} = 4 + j.3 \]

\[ S_{1b} = \frac{V_1 I_1^*}{2} = 4 + j.3 \]

*Difference due to XHR Inductance*
So, we must find a way to model \( Y \) for \( Y_{12} \) as \( Y_a \) varies.

\[
I_1 = Y_{11}V_1 + Y_{12}V_2 \\
I_2 = Y_{12}V_1 + Y_{22}V_2
\]

\[
\hat{S}_1 = -\hat{S}_2 \\
\hat{S}_1 = \frac{V_2}{a} I_1^* \\
S_2 = V_2 I_2^*
\]

\[
\frac{V_2}{a} I_1^* = -V_2 I_2^* \\
I_1^* = -a^* I_2^* \\
I_1 = -a^* I_2
\]

\[
I_1 = (V_1 - \frac{V_2}{a}) Y = \left( Y_{11} - \frac{Y_{12}}{a} \right) V_2 = -a^* I_2
\]

\[
I_2 = -\frac{I_1}{a^*} = -\frac{Y_{11}}{a^*} + \frac{Y_{12}}{a a^*} V_2
\]

\[
Y_{11} = Y \\
Y_{12} = -\frac{Y}{a} \\
Y_{21} = -\frac{Y}{a^*} \\
Y_{22} = \frac{Y}{a a^*} = \frac{Y}{|a|^2}
\]
Paralleling Transformers of Unlike Turns Ratio

\[
\frac{N_1}{N_2} = \frac{V_1}{V_2}
\]

What happens for \(\frac{N_1'}{N_2'} \neq \frac{N_1}{N_2}\)?

Replace \(T_2\) with 2 XFMRS: - First is same ratio as \(T_1\),

\[
\frac{N_1}{N_2} = \frac{N_1'}{X}
\]

\(X = \frac{N_2 N_1'}{N_1}\)

Second XFMR has ratio of off-nominal turns

Per unit equivalent:

\[
\frac{1}{a} = \left(\frac{N_2 N_1'}{N_1'}\right) = \frac{N_2 N_1'}{N_1 N_2}
\]

\[a = \frac{N_1 N_2'}{N_2 N_1'} = \text{p.u. turns ratio}\]

Three Methods to Analyze:
1) Admittance Method
2) Circuit Theory
3) Circulating Current Method (Approximate)
\[ I_a = \frac{1.031 + j0.013}{j1} = .131 - j.311 \]

\[ I_b/a^* = I_2 - I_a = .8 - j.6 - .131 + j.311 = .669 - j.289 \]

\[ S_a = .131 + j.311 \]
\[ S_b = .669 + j.289 \]

Note shift in power flow thru XFMRS

<table>
<thead>
<tr>
<th>BEFORE</th>
<th>AFTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_a )</td>
<td>.4 + j.3</td>
</tr>
<tr>
<td>( I_b/a^* )</td>
<td>.4 - j.3</td>
</tr>
<tr>
<td>( P_a )</td>
<td>.4</td>
</tr>
<tr>
<td>( P_b )</td>
<td>.4</td>
</tr>
<tr>
<td>( Q_a )</td>
<td>+1.3</td>
</tr>
<tr>
<td>( Q_b )</td>
<td>+1.3</td>
</tr>
<tr>
<td>( V_1 )</td>
<td>1.031 12.2°</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>1.0 6°</td>
</tr>
</tbody>
</table>

If \( \alpha \) is at positive angle, power flow through phase shifting XFMRS increases.
\[ S_{1a} = V_2 I_{1a}^* = (0.963 - j4.87)(0.437 - j2.363)^* = 18.76^\circ = \frac{3.98 + j5.35}{41.86} + \frac{4.18 + j3.75}{41.86} \]

### Phase Shifting XFMKR

\[ \alpha = 1 \angle 30^\circ = e^{j30^\circ} \]

**Without phase shifter,**

\[ P_{\text{LOAD}} = \frac{V_2^*}{2} = 1(0.8 + j6) = 0.8 + j6 \]

\[ S_a = 0.4 + j3 \]
\[ S_b = 0.4 - j3 \]
\[ I_a = 0.4 - j3 \]
\[ I_b = 0.4 + j3 \]

**With phase shifter,**

\[ \frac{V_1 - j1}{j1} + \frac{1}{j1} + \frac{1}{j1}(\frac{1}{j1}) = \frac{1}{0.8 + j6} = I_2 \]

\[ V_1(1 + j132^\circ) - 1 = \left(\frac{1}{j132^\circ}\right)^2 = j1(0.8 + j6) \]

\[ V_1(1.999 - j1.5) - 1 = \left(\frac{1.999}{j1.5}\right)^2 = 0.8 + j6 \]

\[ V_1 = 1.031 \angle 173^\circ = 1.031 + j0.13 \approx 1.028 + j0.13 \]

\[ I_2 = 1.028 + j0.13 \]

\[ V_1 = 1.28 \angle 180^\circ = 0 + j0.28 \]

\[ 1 + 0.38 = 1.38 \]
Approximate solution:

Superposition of circulating current

First, look at XFMR before load applied.

\[ I_{\text{circ}} = \frac{0.05/90}{j\cdot 4} = 0.125 \angle -90^\circ = -j0.125 \]

From first example, \( I_{\text{circ}} \)

\[ I_{1a} = 0.4 - j0.3 + j0.125 = 0.4 - j0.125 \]
\[ I_{1b} = 0.4 - j0.3 - j0.125 = 0.4 - j0.425 \]

By superposition,

\[ V_2 = V_2 \text{ before tap change} + \frac{\Delta V(jX_{T1})}{jX_{T1} + jX_{T2}} \frac{Z_{L\text{LOAD}}}{Z_{\text{TOTAL}}} \]

\[ \Delta V = 0.943/4.8^\circ + \frac{0.05(j2)}{j4} \frac{0.8 + j1.5}{0.8 + j0.6} \]

\[ V_2 = 0.963/4.87^\circ \]