EE 5200 - Lecture 24
Fri Oct 22, 2010

Topics for Today:

• Announcements
  • Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  • Office hrs: 2-3pm M,W,F
  • Office: EERC 614. Phone: 906.487.2857
  • Book exercises from Ch.6,7 solutions posted
  • Next homework: Matlab.

Chapter 7 - Network Equations, Admittance Approaches

• How’s your linear algebra? Time to make use of it...
• Basic strategy for building up [Y] for whole network
• Quick recap of xfmrs and lines.
• Generators
• Example of building [Y] for 4-bus system.
• Network Reduction (Kron Reduction)
• Solution of matrix equations (system of linear equations)
• Upcoming homework - intro to Matlab, matrices, equations.
admittance matrix for each of the network branches and then write the nodal

FIGURE 7.3
Single-line diagram of the four-bus system of Example 7.1. Reference node is not shown.

FIGURE 7.4
Reactance diagram for Fig. 7.3. Node (0) is reference, reactances and voltages are in per unit.

\[ y_{34} = y_{43} = 0 \]

"topology"
\[ I_n = \frac{V_{TH}}{Z_{TH}} \]

\[ I_n = V_{TH}Y_n \]

\[ [Z] = [Y]^{-1} \]

\[ [Z] = [Y]^{-1} \]

**FIGURE 7.5**
Per-unit admittance diagram for Fig. 7.4 with current sources replacing voltage sources. If names a to g correspond to the subscripts of branch voltages and currents.

(7.9) applies to each of the other five branches. By setting \( m \) and \( n \) in equations equal to the node numbers at the ends of the individual branches \( c \), we obtain

\[
\begin{bmatrix}
3 & 3 & 3 & 3 & 1 & 4 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & -1 & 1 & 1 \\
-1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
1 & -1 & 1 & 1 \\
\end{bmatrix}
\]

The order in which the labels are assigned is not important here, provided columns and rows follow the same order. However, for consistency with sections let us assign the node numbers in the directions of the branch currents Fig. 7.5, which also shows the numerical values of the admittances. Comuting together those elements of the above matrices having identical row and column labels gives

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & (Y_a + Y_b + Y_f) & -Y_a & -Y_b & -Y_f \\
2 & -Y_a & (Y_a + Y_b + Y_f) & -Y_b & -Y_f \\
3 & -Y_a & -Y_b & (Y_a + Y_b + Y_f) & 0 \\
4 & -Y_f & -Y_f & 0 & (Y_c + Y_d + Y_e)
\end{bmatrix}
\]
\[
\begin{bmatrix}
Y
\end{bmatrix}
\begin{bmatrix}
V
\end{bmatrix} = \begin{bmatrix}
\text{Inj}
\end{bmatrix}
\]

\[
YV = I
\]

\[
[Y]\infty = [Y]^{-1}I
\]

\[
[V] = [Y]^{-1}(I)
\]
[\[Z_{bus}\] - Hard to construct/modify 2
  - Easy to use for S.C. or other S.S. 60-Hz (or 50Hz) calcs.

[\[Y_{bus}\] - Easy to build or modify
  - Less direct in its use for S.C. studies.

Ex: Fig 7.5 in text

\[
\begin{bmatrix}
-14.5 & 8 & 4 & 2.5 \\
8 & -17 & 4 & 5 \\
4 & 4 & -8.8 & 0 \\
2.5 & 5 & 0 & -8.3 \\
\end{bmatrix}
\]

Checks w/ p. 245
Modification is easy:

\[ (-4) \]

ex: Remove Line 2-3:

\[
\begin{bmatrix}
-14 & 5 & 8 & 4 & 2.5 \\
8 & -13 & 0 & 5 \\
4 & 0 & -8 & 10 \\
2.5 & 0 & -8 & 3
\end{bmatrix}
\]

\[ j \]

\[ [Y_{bus}] \text{ is \underline{SPARSE} in general.} \]

Typical "grid" system being analyzed will have 100's or 1000's of buses.

For \( y_{12} = 0 \), no line or xfar from 1-2.

\[ \begin{cases} y_{12} = 0 \\ y_{21} = 0 \end{cases} \]
Typically, only 2-5 buses are connected to a given. i.e., most off-diagonal entries of $[Y]$ are 0. When many entries of a matrix are 0, it's a SPARSE matrix.

- Don't have to store zero values.
  - Single-precision complex values.

For 10,000 bus system:

$\Rightarrow$ 800 MB of RAM.

Use Linked-Link storage, only store the non-zero values.
If each bus is connected to 4 others, 5 then each row has 5 entries.

50,000 non-zero entries
only 400 KB needed.

Import: $[Z] = [Y]^{-1}$ is a full matrix.
Must use factorization methods to obtain desired entries in $[Z] = \ldots$ can find

\[
Z = \begin{bmatrix}
\end{bmatrix}
\]
1. How about if it's a xfer?

2. How to modify [Yours]

3. Turns ratio xfer?

4. Minimum xfer?
...still Only need modify

\[
\begin{bmatrix}
  y_{55} & y_{57} \\
  y_{75} & y_{77}
\end{bmatrix}
\begin{bmatrix}
  V_5 \\
  V_7
\end{bmatrix} =
\begin{bmatrix}
  I_5 \\
  I_7
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
  5 \\
  7
\end{bmatrix}
\]
No phase shift

\[ y = \frac{1}{\sqrt{r + jx}} \]

\[ y_{55} = y_{55} + y \]
\[ y_{71} = y_{71} + y \]
\[ y_{57} = y_{57} - y \]
\[ y_{75} = y_{75} - y \]
\[ y_{55} = y_{55} + y_{5-7} + j \frac{Bc}{2} \]

\[ y_{57} = y_{57} \quad \text{or} \quad -y_{5-7} \]

\[ \begin{bmatrix} y_{55} & y_{57} \\ y_{75} & y_{77} \end{bmatrix} \]

\[ \Sigma \text{Row} 5^- = \frac{\text{Total Shunt}}{\text{Admittance at Bus 5}} \]
In General:

\[ \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

\[ v_1 = \frac{-y_{11} v_1 - y_{12} v_2}{y_{21} v_1 + y_{22} v_2} \]

\[ v_2 = \frac{-y_{21} v_1 - y_{22} v_2}{y_{21} v_1 + y_{22} v_2} \]

\[ y_{11} = \frac{-y_{11} v_1 - y_{12} v_2}{y_{21} v_1 + y_{22} v_2} \]

\[ y_{12} = \frac{-y_{11} v_1 - y_{12} v_2}{y_{21} v_1 + y_{22} v_2} \]

\[ y_{21} = \frac{-y_{21} v_1 - y_{22} v_2}{y_{21} v_1 + y_{22} v_2} \]

\[ y_{22} = \frac{-y_{21} v_1 - y_{22} v_2}{y_{21} v_1 + y_{22} v_2} \]

\[ \text{etc.} \]
\[ V_{TH} = I_N + 2TH \]

\[ Y_n = \frac{1}{2TH} \]

Thevenin's source:

\[ Y = \begin{bmatrix} \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \end{bmatrix}_{mn} \]

\[ [Y][V] = [I_{inj}] \]

\[ [X][X][V_{inj}] = [I_N] \]
\[ Y_{load} = 1.0 - j0.5 \text{pu.} \]

How to add effect into \([Y]\)?

\[
[Y] = \begin{bmatrix}
4.596 \, 1\, -67.97^\circ & 4.64 \, \text{pu} \\
4.64 \, \text{pu} & 5.48 \, 1\, -60.22^\circ
\end{bmatrix}
\]

**Note:** Since load is connected to Bus 2 (Bus 2 - Gnd) then only \(y_{22}\) is affected,

\[ y_{22} \text{ (new)} = y_{22} \text{ (old)} + Y_{load} \]
where

\[ I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a} \quad (7.3) \]

---

**FIGURE 7.1**
Circuits illustrating the equivalence of sources when \( I_s = E_s/Z_a \) and \( Y_a = 1/Z_a \).
Refer to section 2.8 in text...

\[
\begin{align*}
2p_s &= 2p + 2s \\
2p_T &= 2p + 2T \\
2s_T &= 2s + 2T
\end{align*}
\]

\[
\Rightarrow \quad 2p \quad 2s \\
\quad 2T
\]

From transformer nameplate.

Fictitious node, \[Z_s\] is 4x4.

\[
\begin{array}{c}
2p \\
\downarrow \\
2s \\
\downarrow \\
2T \\
\downarrow
\end{array}
\]

Per-Phase "Star" equiv

- All transfer impedances are positive. OK for most S, C, and Load-flow calcs.
- Neg \(Z_s\) can cause trouble in some computer simulations.

When building \([Y]\) for system, be aware!

- \(Z_s\) is often negative, but \(Z_{ps} = Z_p + Z_s\)
- Node in star equivalent does not physically exist.
CAUTION: DO NOT ATTEMPT TO HANDLE, INSTALL, USE OR SERVICE THIS TRANSFORMER BEFORE READING INSTRUCTION BOOK XLL7952-08. TO DO SO MAY LEAD TO BODILY INJURY OR PROPERTY DAMAGE OR BOTH.
Effect of adding 3-wdg xfmr:

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>S</th>
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<tr>
<td>P</td>
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</tbody>
</table>

Note: Above is for Δ-equiv. If using star-equiv, must also add star-point as a new bus.
Example 12 [17]

T-Lines

\[ Y = j \frac{0.08 \text{pu}}{2} \]

Cap: implies that
\[ \frac{B_e}{2} = j0.05 \text{pu} \Rightarrow 2X_c = -j20 \text{pu} \]

\[ Y_{sc} = \frac{1}{2sc} \]

Add a load.
What happens if we attach a load at bus 2?

"LOAD" = 1.0 + j0.5 pu. = S

If we assume load S is given for Y = 1.0 pu, then we can approximate Z load, Y load.
- Large systems, better throughput.

- Sparse, large systems, easy sharing.

800,000
10,000 x 10,000

\[ \text{is better} \]

- Good for four stimuli, hand cards.

\[ \frac{Z_{\text{mm}}}{Z_{\text{II}}} = \frac{Z_{\text{mm}}}{Z_{\text{II}}} \]

\[ Z_{\text{bus}} = \begin{bmatrix} Y_{\text{bus}} \\ Y_{\text{bus}} \end{bmatrix} \]

\[ \text{V.S.} \]

Which is better?

- behavior

[Redacted]