Topics for Today:

- Announcements
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - ASPEN software - run off of MTU server via internet, see e-mail instructions.
  - Office: EERC 614. Phone: 906.487.2857
  - Recommended problems & all solutions: 13 solns now posted.
  - Homework Syst Op - due this Friday, Dec 4th.

Ongoing topics...

- Chapter 13 - Power system operation
  - Constrained optimization methods - LaGrange multipliers
  - Optimal Dispatch, Generator Scheduling
    - Economics
    - Other constraints - environmental, contractual, availability
    - System load characteristics
  - Application to lossless system
  - System including losses - use $[B]$ loss coefficient matrix
Economic Dispatch - Optimum allocation of generation among system generators.

Goal: Maximize system efficiency
Minimize system losses (can't bill customers)

Specifications:
- **Control voltage/vars**
  - Adjust generator exciter
  - Reactors, caps (shunt)
  - Tap-changing transformers

- **Control Power Flow**
  - Control Pgen at each generator
  - Phase-shifting transformers
  - Line switching

- **Frequency** - (later)
  - Prime mover control (droop controller)
  - Load management

\[
P_{GEN} = P_{LOADS} + P_{TRANS/DIST\ LOSSES}
\]

\[
P_G = \sum_{i=1}^{n} P_{Gi}
\]

How should \( P_G \) be divvied up among the \( n \) units?
Constraints:

\( \begin{align*} 
& \text{On-line time regimes - Coal 8 hrs}^+ \\
& \text{Some units down for maintenance} \\
& \text{Should have rolling/rolling reserve in case units fail.} \\
& \quad \begin{cases} 
\quad \text{P}_{\text{min}} < P_i < P_{\text{max}}; \\
\quad \text{Upper constraints} \quad \text{IR of stator of turbine.} 
\end{cases} 
\end{align*} \)

Lingo - Load Characteristics:

A) Daily -

\[ \text{Graph showing daily load characteristics.} \]

B) Weekly -

\[ \text{Graph showing weekly load characteristics.} \]

C) Annual -

\[ \text{Graph showing annual load characteristics.} \]

\[ \text{Graph showing cold and warm climate heat variations.} \]
Load Duration Curve

Load Factor: \( LF = \frac{\text{Energy Used}}{\text{Peak Power} \times \text{hrs}} \)

0.4 - Bad
0.85 - Good

Strategy:

Peak: hydro, gas turbine, pumped storage
Hydro
Small Coal
Base Load - Nukes
Coal

Lose Money
Break Even
Make Money.
Ways a utility can make money:

- Raise rates (PUC/PSC must approve)
- Sell more base load MWH
- Reduce peak load (Load management)
  - Interruptible loads - water heaters, etc
  - Time of day rates
- Increase efficiency
  - Reduce aux use in plant (10-15%)
  - Improve thermal efficiency (Net Heat Rate)

* Economic Dispatch

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For each unit:

\[ HR = \frac{\text{Input Thermal power, Btu/hr}}{\text{Electrical output}} \]

Typical: \[ 10.5 \times 10^6 \text{ Btu/\text{MWh}} \]

- Recognize form as \( V_n \)
But one BTU/hr = 0.293 W

\[ P = \frac{1}{HR \times 0.293 \times 10^{-6}} = \frac{3.413 \times 10^6}{HR} \]

Operating cost of unit i

\[ C_i = F_i \cdot P_i \]

- \( F_i \) Input Power in MBTU (Order of Mag.)
- \( P_i \) Fuel cost in \$/MBTU (\#1.50
  \( + \) labor, supplies, maint).

\[ C_i \text{\$/hr} \]

Empirically,

\[ C_i = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \]

\( \alpha, \beta, \gamma \) in \$/hr
The curve is empirically described as:
\[ C_i = x_i + P_{i1}^2 + x_i P_{i2} + x_i ]

Again,
\[ P_{i1} = P_L + P_{i2} \]

**Problem:** Solve for \( n \) \( P_{i1} \)'s subject to constraints.

Simplest mathematical formulation is to use Lagrange Multipliers.

**Objective function:**
\[ \min F(x_1, x_2, x_3, \ldots, x_n) \]

**Constraints:**
\[ G_1(x_1, x_2, \ldots, x_n) = 0 \]
\[ \vdots \]
\[ G_m(x_1, x_2, \ldots, x_n) = 0 \]

Usually (for our purposes) \( m = 1 \)

1) Form the Lagrangian:
\[ L = F(x_1, x_2, \ldots, x_n) - \lambda G(x_1, x_2, \ldots, x_n) \]

2) Find all partial derivatives of \( L \) wrt \( x_1, x_2, \ldots, x_n \) and set them = 0
3) Solve for \((x_1, x_2, \ldots, x_n)\), \(z\) from partial derivatives \(\nabla G(x_1, x_2, \ldots, x_n)\).

4) Establish whether solution is min/max or saddle point. (Evaluate Hessian Matrix)
   - Min if positive definite
   - Local Max if negative, Saddle if indefinite.

**Example:**

Box of dimensions \(x, y, z\)

Maximize volume for \(S = 432 \text{ cm}^2\)

**Objective function:** \(V = xyz\)

**Constraint:** \(2(xy + 2x^2 + 2xy) - 432 = 0\)

\[
2 = 2x^2y - 7(4x^2 + 6xy - 432)
\]

\[
\frac{\partial V}{\partial x} = 4x - 87x + 6y = 0
\]

\[
\frac{\partial V}{\partial y} = 2x^2 - 63x = 0
\]

**Constraint:** \(2(xy + 2x^2 + 2xy) - 432 = 0\)

Solve simultaneously:

\(x = 2\)

\(y = 8\) cm

\(V = 576 \text{ cm}^3\)
Applying to Economic Dispatch:

Objective: \[ C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i P_{ai}^2 + \beta_i P_{ai} + \gamma_i \]

Constraints: \[ G = P_g - P_L = 0 = \sum_{i=1}^{n} P_{gi} - \sum_{i=1}^{n} P_{li} \]

(ignoring line losses for now.) \[ \text{gen} \] \[ \text{loads} \]

1. \[ \lambda = C - \sum_{i=1}^{n} \left( \alpha_i P_{gi} - \beta_i \right) \]

2. Partial Derivatives:

\[ \frac{\partial \lambda}{\partial P_{gi}} = \frac{\partial C}{\partial P_{gi}} - \lambda (1) = \frac{\partial C}{\partial P_{gi}} - \lambda \]

Since \( \lambda \) is the same in every term, one way to satisfy conditions is:

\[ \frac{\partial C}{\partial P_{gi}} - \lambda = 0, \quad \frac{\partial C}{\partial P_{g2}} - \lambda = 0, \ldots \quad \frac{\partial C}{\partial P_{gen}} - \lambda = 0 \]

Therefore, each plant must be at same incremental cost, \( \lambda_i \) (\( \lambda_1 = \lambda_2 = \ldots = \lambda_n = \lambda \))

For each unit,

\[ \lambda_i = \frac{\partial C_i}{\partial P_{gi}} = 2 \alpha_i P_{gi} + \beta_i \]
Example 8.2

Unit 1: \(25 \text{ MW} < P_{G1} < 150 \text{ MW}\)
\[
C_1 = 0.01 \frac{P_{G1}^2}{\text{MW}} + 2 \frac{P_{G1}}{\text{MW}} + 100
\]

Unit 2: \(30 \text{ MW} < P_{G2} < 200 \text{ MW}\)
\[
C_2 = 0.004 \frac{P_{G2}^2}{\text{MW}} + 2.6 \frac{P_{G2}}{\text{MW}} + 80
\]

How to divide \(P_{G1} + P_{G2}\) within range \(55 \text{ MW} \leq P_L \leq 350 \text{ MW}\) ?

For ex., \(P_L = 282 \text{ MW}\)

First, select \(S_{BASE} = 100 \text{ MVA}\) & convert data to p.u.

\[
\alpha_1 = (100^2)(.01) = 100 \quad \alpha_2 = 40
\]
\[
\beta_1 = (100)(2) = 200 \quad \beta_2 = 260
\]
\[
\gamma_1 = 100 \quad \gamma_2 = 80
\]

\(0.25 \leq P_{G1} \leq 1.50 \text{ p.u.}\)
\(0.30 \leq P_{G2} \leq 2.00 \text{ p.u.}\)
\(0.55 \leq P_L \leq 3.50 \text{ p.u.}\)

\[
\lambda_1 = \frac{\partial C_1}{\partial P_{G1}} = 200 P_{G1} + 200
\]
\[
\lambda_2 = \frac{\partial C_2}{\partial P_{G2}} = 80 P_{G2} + 260
\]
\[
P_{G1} + P_{G2} = 282 \text{ p.u.}
\]
Solving, setting $\lambda_1 = \lambda_2 = \lambda$

$P_{G1} = 1.02$ p.u. (102 MW)  
$P_{G2} = 1.80$ p.u. (180 MW)

Looking at complete range, $55$ MW $\leq P_L \leq 355$ MW

\[ P_{G1} \text{ vs. } P_L \text{ p.u.} \]

At $0.55$ p.u.  
$P_{G1} = 0.25$, $P_{G2} = 0.30$  
$\lambda_1 = 2.50$, $\lambda_2 = 2.84$

must increase unit 1 first, until  
$\lambda_1 = 2.84$. This happens at $P_{G1} = \frac{2.84 - 2.00}{2.00} = 0.42$ p.u.

Then $\lambda_1$ & $\lambda_2$ can be equal until one unit hits $P_{\text{max}}$.  
$\lambda_1 = 5.00$ @ $P_{G1} = 1.5$ p.u.  
$\lambda_2 = 4.20$ @ $P_{G2} = 2.0$ p.u.

$P_{G2}$ limits out first.  
$P_{G1} = \frac{4.20 - 2.00}{2.00} = 1.1$ p.u.

From $P_L = 3.10$ and up, only $P_{G1}$ increases.
Solution

We first select \( S_{3\text{phase}} = 100 \text{ MVA} \); then convert all data into per-unit.

\[
\alpha_1 = (S_{3\text{phase}})^2 = 100 \quad \alpha_2 = 40 \\
\beta_1 = (S_{3\text{phase}})^2 = 200 \quad \beta_2 = 260 \\
\gamma_1 = 100 \quad \gamma_2 = 80
\]

\[
0.25 \leq P_{G_1} \leq 1.50 \text{ pu} \\
0.30 \leq P_{G_2} \leq 2.00 \text{ pu} \\
0.55 \leq P_L \leq 3.50 \text{ pu}
\]

\[
\lambda_1 = \frac{\partial C_1}{\partial P} = 200P + 200 \\
\lambda_2 = \frac{\partial C_2}{\partial P_{G_1}} = 80P_{G_2} + 260
\]

Let us tabulate and plot results as we develop them, as shown in Figure 8.3. We start by calculating \( \lambda_1 \) and \( \lambda_2 \) for minimum-generation conditions (point 1). Observe that \( \lambda_2 > \lambda_1 \). Since we wish to make the \( \lambda \)'s equal, the strategy is to load unit 1 first. We do this until \( \lambda_1 = 284 \), which occurs at

\[
P_{G_1} = \frac{284 - 200}{200} = 0.42 \text{ (point 2)}
\]

Now, calculate \( \lambda_1 \) and \( \lambda_2 \) at the maximum-generation condition (point 3). Observe

\[\lambda_1 = \lambda_2 = 284.5\]

Figure 8.3. Results plotted for example 8.2.

\[\lambda_i > \lambda_j, \text{ suggesting that we unload unit 1 first until we bring } \lambda_1 \text{ down to } \lambda_2 = 420. \text{ This happens at}
\]

\[P_{G_1} = \frac{420 - 200}{200} = 1.10 \text{ (point 4)}
\]

Observe that for \( 0.72 \leq P_L \leq 3.10 \), it is possible to maintain equal \( \lambda \)'s. The equations are

\[\lambda_1 = \lambda_2 \]

\[200P_{G_1} + 200 = 80P_{G_2} + 260\]

and

\[P_{G_1} + P_{G_2} = P_L\]

These linear relations are plotted in Figure 8.3. For \( P_L = 282 \text{ MW} = 2.82 \text{ pu} \),

\[P_{G_2} = 2.82 - P_{G_1}, \quad P_{G_1} = 0.4P_{G_2} + 0.3\]

\[= 1.128 - 0.40P_{G_1} + 0.3\]

\[1.4P_{G_1} = 1.428; \quad P_{G_1} = 1.02 \quad (102 \text{ MW})\]

\[P_{G_2} = 2.82 - 1.02 = 1.80 \quad (180 \text{ MW})\]

Results are presented in Table 8.1.

<table>
<thead>
<tr>
<th>Point</th>
<th>( P_{G_1} )</th>
<th>( P_{G_2} )</th>
<th>( P_L )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.30</td>
<td>0.55</td>
<td>250</td>
<td>284</td>
</tr>
<tr>
<td>2</td>
<td>0.42</td>
<td>0.30</td>
<td>0.72</td>
<td>284</td>
<td>284</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>2.00</td>
<td>3.50</td>
<td>500</td>
<td>420</td>
</tr>
<tr>
<td>4</td>
<td>1.10</td>
<td>2.00</td>
<td>3.10</td>
<td>420</td>
<td>420</td>
</tr>
</tbody>
</table>

In the case where the incremental cost functions \( \lambda_i \) are linearized, a simple straightforward general solution is possible

\[2\alpha_i P_{G_i} - \lambda = -\beta_i \quad i = 1, 2, \ldots, n \quad (8.13a)\]

\[P_{G_1} + P_{G_2} + \cdots + P_{G_n} = P_L \quad (8.13b)\]

\[
\begin{bmatrix}
2\alpha_1 & 0 & 0 & \cdots & 1 \\
0 & 2\alpha_2 & 0 & \cdots & 1 \\
0 & 0 & 2\alpha_3 & \cdots & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & 1 & 1 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
P_{G_1} \\
P_{G_2} \\
P_{G_3} \\
\vdots \\
P_L
\end{bmatrix}
= \begin{bmatrix}
-\beta_1 \\
-\beta_2 \\
-\beta_3 \\
\vdots \\
-P_L
\end{bmatrix} \quad (8.13c)
\]

Solve the linear set for the \( P_{G_i} \)'s and \( \lambda \).