Topics for Today:

• Announcements
  • Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  • Office: EERC 614. Phone: 906.487.2857
  • Recommended problems & all solutions: 13 solns now posted.
  • Homework Syst Op - due Dec 6th.

Ongoing topics, wrapping up optimal dispatch...

Chapter 16 - Stability
• Dr. Mork’s lecture notes “System Stability” – See Week 13.
• Basic overview. Lead-in to EE6210 (Kundur’s taxonomy).
  • Angle stability vs. voltage stability
  • Small disturbance vs. Transient stability
  • H: Stored energy per MW, J: rotational moment of inertia
  • Coherency
  • Swing Equation
  • Equal area review
  • Reclosing strategies
Before, without losses: \( P_G = \sum_{i=1}^{n} P_{Gi} = P_L \)

With losses: 
\[
P_G = P_L + P_{TL}
\]

Constraint Equation: 
\[
G = P_G - P_L - P_{TL} = 0
\]

Main problem is to find \( P_{TL} \).

From Kron, Kirchmeyer, George:

Losses in terms of \( P_{Gi} \): 
\[
P_{TL} = \sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij} P_{Gj}
\]

\[
= [P_G]^T [B] [P_G]
\]

\[
= [P_{G1}, P_{G2}, \ldots, P_{Gn}] [B] [P_{G1}, P_{G2}, \ldots, P_{Gn}]
\]

How to determine \( B \) constants?

(Many methods, Each utility to their own.)

1) Empirical Method: "Hill-Stevenson Method"

Run lots of load flows and examine sensitivity of all \( B \)'s to \( P_{Gi} \)

Key: Find all 
\[
\frac{\partial S_k}{\partial P_{Gi}} \bigg|_{k=1, n}
\]

Refer to method in book.
Once $B$ coefficients are obtained, setting up the Lagrangian:

**minimize:** $\sum_{i=1}^{n} C_i$ \hspace{1cm} \text{ (objective) }

**subject to:** $P_G - P_L - P_{TL} = 0$ \hspace{1cm} \text{ (constraint) }

$\mathcal{L} = \sum_{i=1}^{n} C_i - \lambda (P_G - P_L - P_{TL})$

\[ \frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial C_i}{\partial P_{Gi}} - \lambda \left(1 - \frac{\partial P_{TL}}{\partial P_{Gi}}\right) = 0 \]

For optimized condition, $\lambda = \lambda_i$ \hspace{1cm} (all $\lambda$s equal)

Frame before, \[ \frac{\partial C_i}{\partial P_{Gi}} = 2 \lambda_i P_{Gi} + \beta_i \]

\[ \frac{\partial P_{TL}}{\partial P_{Gi}} = 2 \sum_{j=1}^{n} B_{ij} P_{Gj} \]

**Constraint Equation:**

\[ \sum_{i=1}^{n} P_{Gi} - P_L - \sum_{i=1}^{n} \sum_{j=1}^{n} P_{Gi} B_{ij} P_{Gj} = 0 \]
Once $[B]$ is known:

\[ PF_i = \frac{1}{1 - 2 \sum_{j=1}^{n} B_{ij} P_{ij}} = \frac{\partial P_{ii}}{\partial P_{ii}} \text{ Penalty Factor} \]

\[ 2 i = \frac{\partial C_i}{\partial P_{ii}} \]

\[ \frac{\partial C_i}{\partial P_{ii}} = 2 \alpha_i P_{ii} + \beta_i \]

\[ PF_i = 1 - \frac{\partial P_{ii}}{\partial P_{ii}} \]

\[ 2 i = \left( \frac{\partial C_i}{\partial P_{ii}} \right) \left( PF_i \right) \]

\[ \left( 1 - \frac{\partial P_{ii}}{\partial P_{ii}} \right) 2 i = \frac{\partial C_i}{\partial P_{ii}} \]
For optimum, \( a = 3 \); (all \( a \)'s the same)

\[
\lambda_i = \left( \frac{\partial C_i}{\partial P_{ai}} \right) (PF_i) = \lambda_i^{(sys)}
\]

\( \lambda_i = \frac{2a_i P_{gi} + \beta_i}{1 - 2 \sum_{j=1}^{n} \beta_{ij} P_{gj}} \)

\[
[B] = \begin{bmatrix}
0.01694 & -0.004940 \\
-0.004940 & 0.014406
\end{bmatrix}
\]

\[
\lambda_1 = \frac{2a_1 P_{g1} + \beta_1}{1 - 2 (B_{11} P_{g1} + B_{12} P_{g2})}
\]

\( \lambda_1 = 100 \)

\( \beta_1 = 200 \)

\( \gamma = ? \)

\[
\lambda_2 = \frac{2a_2 P_{g2} + \beta_2}{1 - 2 (B_{21} P_{g1} + B_{22} P_{g2})}
\]

\( \lambda_2 = 40 \)

\( \beta_2 = 260 \)
\[ T_1 = \frac{200 P_{G1} + 260}{1 - 2\left[ (0.00694) P_{G1} - (0.0049) P_{G2} \right]} \]

\[ T_2 = \frac{80 P_{G2} + 260}{1 - 2\left[ (-0.00494) P_{G1} + (0.014406) P_{G2} \right]} \]

\[ P_{TL} = B_{11} P_{G1}^2 + 2 B_{12} P_{G1} P_{G2} + B_{22} P_{G2}^2 \]

\[ P_{G1} + P_{G2} - P_{TL} = 2.82 \]

Solve trial & error (or could set up N-R).

(Could use starting values from case where line losses are neglected.)

i.e. - Starting values of \( P_{G1} \) & \( P_{G2} \)

Start: \( P_{G1} = 1.02 \text{ p.u.} \), \( P_{G2} = 1.80 \text{ p.u.} \)

then calculate \( T_1 \), \( T_2 \), \( P_{TL} \).

If \( T_2 > T_1 \), then decrease \( P_{G2} \), increase \( P_{G1} \)

Calculate \( P_{TL} \) & \( P_{G1} + P_{G2} - P_{TL} \).

If \( P_{TL} < 2.82 \), increase both \( P_{G1} \) & \( P_{G2} \).

See table on page 328.

Hand calculation recommended to give students a feel for problem & interrelationships.
8.2 Economic Dispatch Considering Losses

In section 8.1, the economic dispatch problem was solved neglecting transmission losses. Experience has shown that in some cases, this approximation produces results that are in serious error. We recall the power balance equation

\[ P_g = P_L + P_{TL} \]  

(8.7a)

The revised equation of constraint, considering losses, is then

\[ G = P_g - P_L - P_{TL} = 0 \]  

(8.14)

Recall that the problem’s variables are the \( P_g \)'s. Therefore, it is necessary to formulate the losses using the \( P_g \)'s as variables.

Let us consider the losses of the components of the transmission system, specifically, transformers and lines. Transformers have two types of losses: copper and iron. The iron, or magnetic, losses vary with core flux density, which in turn varies with voltage. Since the transformer voltage varies indirectly, and very little, with load, the variation of core loss with load is quite small and not important. However, the copper loss varies as \( I^2R \); the load current varies directly with power loading; and the series resistance loss varies with the square of this current. Thus, the transformer copper loss should be included in the transmission losses. Likewise, the \( I^2R \) losses in the series element of the transmission line constitute transmission losses. The challenge is to functionally relate these \( I^2R \) losses to the \( P_g \) variables.

Many investigators, including Kron, Kirchmeyer, and George have worked on this problem and proposed a loss equation that formulates the loss as a quadratic function of the \( P_g \)'s;†

\[ P_{TL} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{gi} B_{ij} P_{gj} \]  

(8.15a)

\[ = \mathbf{P}_g^{\top} \mathbf{B} \mathbf{P}_g \]  

(8.15b)

where

\[ \mathbf{P}_g = \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn} \end{bmatrix} = (n \times 1) \quad \text{vector of generated powers.} \]

\[ \mathbf{B} = (n \times n) \quad \text{array with general entry } B_{ij}. \]

The \( \mathbf{B} \) array is symmetrical, such that \( B_{ij} = B_{ji} \). Computing the \( B_{ij} \) values, called the \( B \) constants, can be implemented by several techniques. One approach, called the Hill–Stevenson method, calculates \( B \) constants using partial derivatives,

\[ \frac{\partial P_{TL}}{\partial P_{gk}} = 2 \sum_{i=1}^{n} B_{ik} P_{gj} \]  

(8.16a)

\[ = 2 \sum_{i=1}^{n} B_{ij} P_{gj} \]  

(8.16b)

Furthermore,

\[ \frac{\partial^2 P_{TL}}{\partial P_{gk} \partial P_{gl}} = 2B_{lm} = 2B_{mk} \]  

(8.17a)

But the subscript notation is arbitrary, so that

\[ B_{ij} = \frac{1}{2} \left( \frac{\partial^2 P_{TL}}{\partial P_{gk} \partial P_{gl}} \right) \]  

(8.17b)

Consider the general power system as we viewed it for power flow in Figure 7.2. The total power injected into the transmission network is†

\[ P_{TL} = \text{Re} \left[ \sum_{i=1}^{b} V_i I_i^{\ast} \right] \]  

(8.18a)

If the transmission network were lossless, \( P_{TL} \) would be zero (power in = power out); in general, this summation would equal the total transmission system loss \( P_{TL} \).

\[ P_{TL} = \text{Re} \left[ \sum_{i=1}^{b} V_i \left( \sum_{j=1}^{b} V_j y_{ij} \right)^{\ast} \right] \]  

(8.18b)

\[ = \sum_{i=1}^{b} \sum_{j=1}^{b} V_i V_j y_{ij} \cos(\delta_i - \delta_j - \gamma_{ij}) \]  

(8.18c)

Now, we compute from equation (8.18c)‡:

\[ \frac{\partial P_{TL}}{\partial \delta_k} = 2 \sum_{i=1}^{b} V_i V_k y_{ik} \sin(\delta_i - \delta_k) \]  

(8.19)

Now, by the chain rule, consider that

\[ \frac{\partial P_{TL}}{\partial P_{gi}} = \sum_{k=1}^{b} \frac{\partial P_{TL}}{\partial \delta_k} \frac{\partial \delta_k}{\partial P_{gi}} \]  

(8.20)

† Here, \( b \) is the total number of buses, and \( n \) generators. In Chapter 7, \( n \) was the total number of\n
‡ The simplification is complicated and relegated to an exercise for the student. See problem 8.7.
316 8 Operating and Controlling Power Systems

The troublesome terms are the $\partial \delta_i/\partial P_{G_j}$ factors. The Hill-Stevenson method provides a clever way of evaluating these partial derivatives numerically. A base load flow case is established that hopefully approximates the ultimate allocation of generation. One approach is to solve the lossless economic dispatch problem as a first approximation. Then increase all bus loads proportionally by a small amount (say, 10%), and allow generator 1 alone to pick up the increased load. The load flow solves for the phase ($\delta_i$) at all b buses, from which the change in phase ($\Delta \delta_i$) can be calculated. Therefore,

$$\frac{\partial \delta_i}{\partial P_{G_j}} \equiv \frac{\Delta \delta_i}{\Delta P_{G_j}} = A_{kj}$$

This partial derivative is reasonably constant over a rather wide range of generation and load values. We repeat the process n times, allowing each generator to act alone to pick up the increase in load, such that

$$\frac{\partial \delta_i}{\partial P_{G_j}} \equiv \frac{\Delta \delta_i}{\Delta P_{G_j}} = A_{kj} \quad k = 1, 2, \ldots, b \quad j = 1, 2, \ldots, n$$ (8.21)

The partial derivatives so approximated are represented as the $A$ constants. Let us now compute a second partial derivative

$$\frac{\partial}{\partial P_{G_i}} \left( \frac{\partial P_{TL}}{\partial P_{G_j}} \right) = \frac{\partial^2 P_{TL}}{\partial P_{G_i} \partial P_{G_j}} = \frac{\partial}{\partial P_{G_j}} \left[ \sum_{k=1}^{b} \frac{\partial P_{TL}}{\partial \delta_k} \left( \frac{\partial \delta_k}{\partial P_{G_i}} \right) \right]$$ (8.22a)

$$= \sum_{m=1}^{b} \sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} \left( \frac{\partial \delta_k}{\partial P_{G_i}} \right) \left( \frac{\partial \delta_m}{\partial P_{G_j}} \right)$$

$$= \sum_{m=1}^{b} \sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} \left( \frac{\partial \delta_k}{\partial P_{G_i}} \right) \left( \frac{\partial \delta_m}{\partial P_{G_j}} \right)$$

or

$$\frac{\partial^2 P_{TL}}{\partial P_{G_i} \partial P_{G_j}} = \sum_{m=1}^{b} \sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} A_{kj} A_{mi}$$ (8.22b)

Recall that

$$\frac{\partial P_{TL}}{\partial \delta_k} = 2 \sum_{i=1}^{b} V_i \cos(\delta_i - \delta_k)$$ (8.19)

We wish to compute

$$\frac{\partial}{\partial \delta_m} \left( \frac{\partial P_{TL}}{\partial \delta_k} \right) = \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k}$$ (8.23a)

where $\delta_m$ is one specific $\delta_i$. We determine that

$$\frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} = 2V_m V_k \theta_{mk} \cos(\delta_m - \delta_k) \quad m \neq k$$ (8.23b)

$$\frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} = -2 \sum_{i=1}^{b} V_i V_m \theta_{im} \cos(\delta_i - \delta_m) \quad m = k$$ (8.23c)

To relate this work to the $B$ constants, recall that

$$B_{ij} = \frac{1}{2} \left( \frac{\partial^2 P_{TL}}{\partial P_{G_i} \partial P_{G_j}} \right)$$ (8.24)

Finally,

$$B_{ij} = \frac{1}{2} \left( \sum_{m=1}^{b} \sum_{k=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} A_{mi} A_{kj} \right)$$ (8.25)

We now return to our main concern, that is, solving the economic dispatch problem considering losses.

$$C = \sum_{i=1}^{n} C_i \quad \text{(objective function)}$$ (8.6)

$$P_G - P_L - P_{TL} = 0 \quad \text{(equation of constraint)}$$ (8.14)

$$\mathcal{L} = \sum_{i=1}^{n} C_i - \lambda [P_G - P_L - P_{TL}] \quad \text{(the Lagrangian)}$$ (8.26)

The general partial derivative with respect to $P_{G_i}$ is

$$\frac{\partial \mathcal{L}}{\partial P_{G_i}} = \frac{\partial C_i}{\partial P_{G_i}} - \lambda \left( 1 - \frac{\partial P_{TL}}{\partial P_{G_i}} \right) = 0$$ (8.27)

or, we redefine $\lambda_i$ such that

$$\lambda_i = \frac{\partial C_i}{\partial P_{G_i} (\partial P_{TL} / \partial P_{G_i})} = \lambda$$ (8.28a)

where

$$\frac{\partial C_i}{\partial P_{G_i}} = 2\alpha_i P_{G_i} + \beta_i$$ (8.28b)

and

$$\frac{\partial P_{TL}}{\partial P_{G_i}} = 2 \sum_{j=1}^{n} B_{ij} P_{G_j}$$ (8.28c)
and the equation of constraint requires that

\[
\left( \sum_{i=1}^{n} P_{e_i} \right) - P_{L} - \left( \sum_{i=1}^{n} \sum_{j=1}^{n} P_{e_i} B_{ij} P_{e_j} \right) = 0
\]  

(8.29)

Thus, the condition for economically optimum operation requires the weighted incremental cost functions for all units to be equal (to each other and \( \lambda \))

\[
\lambda_i = \lambda
\]  

(8.30)

The weighting factor is sometimes called the penalty factor of generator \( i(PF_i) \)

\[
\lambda_i = \frac{1}{1 - (\partial P_{TL}/\partial P_{e_i})}
\]  

(8.31)

Example 8.3 is useful for illustrative purposes.

**Example 8.3**

A single-line diagram for the system in example 8.2 is shown in Figure 8.4. A base case load flow study on the system provides the following results:

\[
\begin{align*}
V_1 &= 1/0^\circ & P_{e_1} &= 1.0313 & P_{L_1} &= 2.1000 \\
V_2 &= 1/6.616^\circ & P_{e_2} &= 1.8200 & P_{L_2} &= 0.7200 \\
& & P_{TL} &= 0.0313
\end{align*}
\]

\[
\begin{align*}
\bar{X} &= 0.025 + j0.100 \\
\bar{Y}/2 &= 0.050 \\
\bar{X}_1 &= 2.10 + j0 \\
\bar{Y}_2 &= 0.72 + j0
\end{align*}
\]

All values in per-unit

**Figure 8.4.** System for example 8.3.

(a) Calculate the \( A \) constants. The load at each bus was increased 10%. If unit 1 picks up the load,

\[
\Delta \delta_1 = 0 \text{ (using bus 1 as phase reference)}
\]

\[
\Delta \delta_2 = 6.187 - 6.616 = -0.429^\circ \text{ (} -0.007487 \text{ rad)}
\]

\[
\Delta P_{e_1} = 1.3094 - 1.0313 = 0.2781
\]

\[
A_{11} = 0 \quad A_{21} = \frac{-0.007487}{0.278100} = -0.026924
\]

(b) Calculate the \( B \) constants

\[
[Y] = \begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{align*}
\theta_{11} &= \theta_{22} = 2.353 \\
\theta_{12} &= \theta_{21} = -2.353
\end{align*}
\]

\[
m = k
\]

\[
\frac{1}{2} (\partial^2 P_{TL}/\partial \delta_m \partial \delta_k) = - \sum_{m=1}^{2} \sum_{k=1}^{2} \bar{V}_i \bar{V}_k \theta_{m} \cos(\delta_i - \delta_m)
\]

\[
= -(1)(1) \cos(0 - 6.616^\circ)
\]

\[
= +2.337
\]

\[
m \neq k
\]

\[
\frac{1}{2} (\partial^2 P_{TL}/\partial \delta_m \partial \delta_k) = \bar{V}_m \bar{V}_k \theta_{m} \cos(\delta_m - \delta_k)
\]

\[
= (1)(-2.337) = -2.337
\]

Finally,

\[
B_{ij} = \sum_{m=1}^{2} \sum_{k=1}^{2} \frac{1}{2} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_k} A_{m} A_{k}
\]

\[
= 2.337(A_{11}A_{11} - A_{11}A_{21} - A_{21}A_{11} + A_{21}A_{21})
\]

\[
B_{11} = 2.337[(-0.026924)^2] = 0.001694
\]

\[
B_{12} = 2.337[(-0.026924)(-0.0078507)] = -0.004940
\]

\[
B_{22} = 2.337[(-0.0078507)^2] = 0.014406
\]

Check \( P_{TL} \)

\[
P_{TL} = B_{11} P_{e_1}^2 + 2B_{12} P_{e_1} P_{e_2} + B_{22} P_{e_2}^2
\]

\[
= (0.001694)(1.0313)^2 - 2(0.004940)(1.0313)(1.82) + (0.014406)(1.82)^2
\]

\[
= 0.0310 \text{ (compared with 0.0313; the small error is due to the fact that the small angle changes were in the order of the load flow convergence criteria).}
\]
(c) Solve for the economically optimum division of load considering losses. The penalty factors are

\[
P F_1 = \frac{1}{1 - (\partial P_{TL}/\partial P_{G1})} = \frac{1}{1 - 0.003388P_{G1} + 0.009881P_{G2}}
\]

\[
P F_2 = \frac{1}{1 + 0.009881P_{G1} - 0.028811P_{G2}}
\]

\[
\lambda_1 = 200(P_{G1} + 1)
\]

\[
\lambda_2 = \frac{80P_{G2} + 260}{1 + 0.009881P_{G1} - 0.028811P_{G2}}
\]

The problem is solved by trial and error using a programmable calculator.

The results of three trials are

<table>
<thead>
<tr>
<th>( P_{G1} )</th>
<th>( P_{G2} )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( P_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0313</td>
<td>1.8200</td>
<td>400.4</td>
<td>423.5</td>
<td>2.820</td>
</tr>
<tr>
<td>1.1100</td>
<td>1.7400</td>
<td>416.4</td>
<td>415.5</td>
<td>2.823</td>
</tr>
<tr>
<td>1.1060</td>
<td>1.7410</td>
<td>415.6</td>
<td>415.6</td>
<td>2.820</td>
</tr>
</tbody>
</table>

The general problem is too complex to solve by hand. A flow chart for computer implementation is shown in Figure 8.5. The problem may also be formulated and solved by Newton-Raphson methods.

The foregoing discussion applies basically to an all-thermal system; that is, one where all generating units can be assigned an operation cost function \( C_i \). If hydro units are available, there is no fuel cost associated with the unit output and hence a negligible \( C_i \) function. However, this does not mean that there are no restrictions on hydro output whatsoever. The typical restriction involves the volume of water that may be removed from a reservoir in a specific time period. Consider

\[
P_H = \eta H g \rho \frac{dv}{dt} \quad (8.32a)
\]

where

\( P_H \) = hydraulic turbine power in W.
\( \eta \) = hydraulic efficiency of turbine and penstock.
\( g \) = acceleration of gravity = 9.8 m/s².
\( H \) = difference in elevation of reservoir surface and turbine (head) in m.
\( \rho \) = mass density of water = 1000 kg/m³
\( \frac{dv}{dt} \) = volume of water flow through turbine in m³/s.

For a constant-head, constant-efficiency situation,

\[
P_H = K \frac{dv}{dt} \quad (8.32b)
\]

\[
K = \eta H g \rho \quad (8.32c)
\]

so that

\[
\int dv = \frac{1}{K} \int P_H dt \quad (8.33a)
\]
Power System Stability

- Ability to remain in operating equilibrium
- Equilibrium between opposing forces

Angle Stability
- Ability to maintain synchronism
- Torque balance of synchronous machines

Voltage Stability
- Ability to maintain steady acceptable voltage
- Reactive power balance

Large-Disturbance Voltage Stability
- Large disturbance
- Switching events
- Dynamics of ULTC, loads
- Coordination of protections and controls

Transient Stability
- Large disturbance
- First-swing aperiodic drift
- Study period up to 10 s

Mid-term Stability
- Severe upsets; large voltage and frequency excursions
- Fast and slow dynamics
- Study period to several min.

Long-term Stability
- Uniform system frequency
- Slow dynamics
- Study period to tens of min.

Small-Signal Stability

Non-oscillatory Instability
- Insufficient synchronizing torque

Oscillatory Instability
- Insufficient damping torque
- Unstable control action

Local Plant Modes
Interarea Modes
Control Modes
Torsional Modes

* With availability of improved analytical techniques providing unified approach for analysis of fast and slow dynamics, distinction between mid-term and long-term stability has become less significant.

Figure 2.9 Classification of power system stability
\[ P_{1 \rightarrow 2} = \frac{V_1 V_2 \sin(\delta_1 - \delta_2)}{x} \]

\[ Q_{1 \rightarrow 2} = \frac{V_1 V_2}{x} \cos(\delta_1 - \delta_2) - \frac{V_2^2}{x} \]

\[ P: \text{most sensitive to } \delta. \]

\[ Q: \text{most sensitive to } V. \]

Stability Swing: typically 1-2 Hz.

\[ V_1 L S_1 \]

\[ V_2 L S_2 \]