EE 5200 - Lecture 24 (From 10/22-2010) Fri Oct 21, 2011

Topics for Today:

• Announcements
  • Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  • Office hrs: 2-3pm M,W,F
  • Office: EERC 614. Phone: 906.487.2857
  • Book exercises from Ch.6,7 solutions posted
  • Next homework: Matlab.

Chapter 7 - Network Equations, Admittance Approaches

• How’s your linear algebra? Time to make use of it...
• Basic strategy for building up $[Y]$ for whole network
• Quick recap of xfmrs and lines.
• Generators
• Example of building $[Y]$ for 4-bus system.
• Network Reduction (Kron Reduction)
• Solution of matrix equations (system of linear equations)
• Upcoming homework - intro to Matlab, matrices, equations.
admittance matrix for each of the network branches and then write the nodal
\[ I_N = \frac{V_{TH}}{Z_{TH}} \]

\[ I_N = V_{TH} Y_N \]

\[ [Z] = [Y]^{-1} \]

\[ \begin{bmatrix}
  Y & \vdots & \vdots & \vdots \\
  \vdots & \ddots & \vdots & \vdots \\
  \vdots & \vdots & Y & \vdots \\
  \vdots & \vdots & \vdots & Y
\end{bmatrix} \begin{bmatrix}
  1 \\
  -1 \\
  1 \\
  -1
\end{bmatrix} \begin{bmatrix}
  Y & \vdots & \vdots & \vdots \\
  \vdots & \ddots & \vdots & \vdots \\
  \vdots & \vdots & Y & \vdots \\
  \vdots & \vdots & \vdots & Y
\end{bmatrix}^{-1} = \begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix} \]

Figure 7.5
Per-unit admittance diagram for Fig. 7.4 with current sources replacing voltage sources. The
names e to g correspond to the subscripts of branch voltages and currents.

(7.9) applies to each of the other five branches. By setting m and n in equations equal to the node numbers at the ends of the individual branches C, we obtain

\[ \begin{bmatrix}
  3 \\
  3 \\
  3 \\
  4
\end{bmatrix} [1] Y_e \begin{bmatrix}
  2 & [-1 & 1] Y_e & 1 & [-1 & 1] Y_e \\
\end{bmatrix} \begin{bmatrix}
  3 \\
  3 \\
  3 \\
  4
\end{bmatrix} \]

The order in which the labels are assigned is not important here. Provide columns and rows follow the same order. However, for consistency with sections let us assign the node numbers in the directions of the branch currents Fig. 7.5, which also shows the numerical values of the admittances. Combining those elements of the above matrices having identical row and column labels gives

\[ \begin{bmatrix}
  1 & \begin{bmatrix}
    (Y_e + Y_y + Y_f) & -Y_e & -Y_e & -Y_f \\
    -Y_e & (Y_e + Y_y + Y_f) & -Y_y & -Y_f \\
    -Y_e & -Y_y & (Y_e + Y_y + Y_f) & 0 \\
    -Y_f & -Y_f & 0 & (Y_e + Y_f + Y_y + Y_f)
  \end{bmatrix}
\end{bmatrix} \]
\[ [Z_{bus}] \text{ - Hard to construct/modify} \]
\[ [Y_{bus}] \text{ - Easy to build or modify} \]
\[ [Y_{bus}] \text{ - Less direct in its use for S.C. studies.} \]

Ex: Fig 7.5 in text

\[
[Y_{bus}] = j \begin{bmatrix}
-14.5 & 8 & 4 & 2.5 \\
8 & -17 & 4 & 5 \\
4 & 4 & -8.8 & 0 \\
2.5 & 5 & 0 & -8.3
\end{bmatrix}
\]

Checks w/ p. 245
Modification is easy:
ex: Remove Line 2-3: \((-jt)\)

\[
\begin{bmatrix}
-14.5 & 8 & 4 & 2.5 \\
8 & -13 & 0 & 5 \\
4 & \frac{-0}{5} & -4.810 \\
2.5 & \frac{-0}{5} & 0 & -8.3
\end{bmatrix}
\]

\([Ybus]\) is SPARSE in general.

Typical "grid" system being analyzed will have 100's or 1000's of buses.

\[
\begin{cases}
y_{12} = 0, \text{ no line or xfar from 1-2.} \\
y_{21} = 0
\end{cases}
\]
Typically, only 2-5 buses are connected to a given. i.e., most off-diagonal entries of [Y] are 0.

When many entries of a matrix are 0, it's a SPARSE matrix.

- Don't have to store zero values.
  Single-precision complex values, 8 bytes.

For 10,000 bus system:

\[ \Rightarrow \text{800 MB of RAM.} \]

Use Linked-Link storage, only store the non-zero values.
If each bus is connected to 4 others, 5 then each row has 5 entries.

\[ \Rightarrow \text{50,000 non-zero entries} \]

\[ \text{only 400 KB needed.} \]

\[ \text{Import: } [E] = [Y]^T \text{ is a full matrix.} \]

Must use factorization methods to obtain desired entries in 
\[ [E] \Rightarrow \text{can find} \]

\[ z = \begin{bmatrix}
\end{bmatrix} \]
- How to modify [Ybus]?  
- How about if it's a xfmr?  
- ""  ""  ""  "" off-nominal turns ratio xfmr?
\[
\begin{bmatrix}
y_{55} & y_{57} \\
y_{75} & y_{77}
\end{bmatrix}
\begin{bmatrix}
y_5 \\
y_7
\end{bmatrix}
= \begin{bmatrix}
y_{55} & y_{57} \\
y_{75} & y_{77}
\end{bmatrix}
\begin{bmatrix}
y_5 \\
y_7
\end{bmatrix}
\]

\[\Rightarrow \]

\[\begin{bmatrix}
y_5 \\
y_7
\end{bmatrix}
\]

\[\begin{bmatrix}
y_{55} & y_{57} \\
y_{75} & y_{77}
\end{bmatrix}
\]

\[\Rightarrow \]

\[\begin{bmatrix}
y_5 \\
y_7
\end{bmatrix}
\]

.....still Only need modify

\[\begin{bmatrix}
y_{55} & y_{57} \\
y_{75} & y_{77}
\end{bmatrix}
\]
No phase shift

\[
R \begin{bmatrix}
  y \\
  -y \\
-\gamma \\
y
\end{bmatrix}
\]

\[
y_{55} = y_{55} + y
\]
\[
y_{71} = y_{71} + y
\]
\[
y_{57} = y_{57} - y
\]
\[
y_{75} = y_{75} - y
\]
\[ y_{55} = y_{SS} + y_{5-7} + j \frac{Bc}{2} \]

\[ y_{57} = y_{SS} + y_{5-7} \]

\[ \begin{bmatrix} y_{55} & y_{57} \\ y_{75} & y_{77} \end{bmatrix} \]

\[ \sum \text{Row 5} = \frac{\text{Total Shunt}}{\text{Admittance at Bus 5}} \]
2. Winding Xffw

(0° phase shift)

$\frac{x^2}{(B_s)(A)}$

Bus 7

$y \cdot \frac{1}{3t}$

$y + \frac{1}{3t}$

$y + \frac{1}{2t}$

10-bus System

$5a = \phi$
In General:

\[ \begin{bmatrix} \bar{y}_{11} & \bar{y}_{12} \\ \bar{y}_{21} & \bar{y}_{22} \end{bmatrix} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix} \]

\[ \bar{y}_{11} = \frac{\bar{v}_1}{\bar{i}_1}, \quad \bar{y}_{12} = \frac{\bar{v}_1}{\bar{i}_2}, \quad \bar{y}_{21} = \frac{\bar{v}_2}{\bar{i}_1}, \quad \bar{y}_{22} = \frac{\bar{v}_2}{\bar{i}_2} \]

With:

\[ \bar{v}_1 = 0 \]

\[ \bar{v}_2 = 0 \]

etc.
Thevenin's Source

\[ Y_n = \frac{1}{2} Y_{TH} \]

\[ V_{TH} = I_N + 2\cdot Y_{TH} \]

\[
\begin{bmatrix}
Y
\end{bmatrix}
\begin{bmatrix}
V
\end{bmatrix} =
\begin{bmatrix}
I_{inj}\end{bmatrix}
\]

\[
\begin{bmatrix}
x
\end{bmatrix}
\begin{bmatrix}
Y_{inj}
\end{bmatrix} =
\begin{bmatrix}
I_N\end{bmatrix}
\]
\[
Y_{zz} = Y_{zz} \text{ (new)} + Y_{load} = Y_{zz} \text{ (old)} + Y_{load}
\]

Note: Since load is connected to Bus 2 (Bus 2 - Ground), then only \( Y_{zz} \) is affected.

\[\begin{bmatrix}
5.48(55.1) \\
4.64(111.8) \\
\end{bmatrix} \rightarrow \begin{bmatrix}
4.64(111.8) \\
4.596(1-67.97) 4.64(111.8) \\
\end{bmatrix} \]

\[Y = [Y] + \text{Load} = 1.0 - 1.0 \text{ p.u.} \]

\[\vdots\]
where

\[ I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a} \]  \hspace{1cm} (7.3)

**FIGURE 7.1**
Circuits illustrating the equivalence of sources when \( I_s = \frac{E_s}{Z_a} \) and \( Y_a = \frac{1}{Z_a} \).
3-Winding XFMRs

See Section 2.8 in text.

NAME PLATE EX.

(See last page.)

EACH LEG OF CORE

H2, I2

H1, I1

P, S

A, C, D

H1, I1, H2, I2
Refer to section 2.8 in text...

\[ Z_{ps} = Z_p + Z_s \]
\[ Z_{pt} = Z_p + Z_T \]
\[ Z_{st} = Z_s + Z_T \]

\[ \Rightarrow \]
\[ \frac{Z_p}{Z_s} \]
\[ \frac{Z_s}{Z_T} \]

Fictitious node, \( \mathcal{N} \) is 4 \times 4.

Per-Phase "Star" equiv

- All transfer impedances are positive. OK for most S, C, and Load-flow calc.
- Neg. \( Z_s \) can cause trouble in some computer simulations.

When building \( [Y] \) for system, be aware!

- \( Z_s \) is often negative, but \( Z_{ps} = Z_p + Z_s \) is always positive.
- Node in star equivalent does not physically exist.

From transformer nameplate.

Can convert \( Y \Rightarrow \Delta \)

\[ P \]
\[ \Rightarrow \]
\[ \begin{bmatrix} 3 \times 3 \end{bmatrix} [Y]. \]
**CAUTION:**
DO NOT ATTEMPT TO HANDY, INSTALL, USE OR SERVICE THIS TRANSFORMER BEFORE READING INSTRUCTIONS BORE XLL7952-13. TO DO SO MAY LEAD TO BODILY INJURY OR PROPERTY DAMAGE ON BOTH.

**NOTE:**
This is a repair of existing equipment. Transformer parts repaired, XLL7952-13.

**VOLTAGE AND CURRENT RATINGS FOR THIS TRANSFORMER:**

<table>
<thead>
<tr>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
</tr>
</thead>
<tbody>
<tr>
<td>102200</td>
<td>88100</td>
<td>127200</td>
</tr>
</tbody>
</table>

**TEST LEVEL:**
H-Secondary 650 KV, X-Secondary 250 KV.
H-Neutral, 110 KV, Tertiary 110 KV.

**REPAIRED IN ST. LOUIS MO. U.S.A.**

**NP# XLL7952-10 SUB A**
**Effect of adding 3-wdg xfmr:**

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>S</th>
<th>T</th>
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<tbody>
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</tbody>
</table>

_Above is for Δ-equiv._

If using star-equiv, must also add star-point as a **new** bus.
T-Lines

Example:
Build [4].

\[ R_e = \omega C \]
\[ Y = jB_e \]

Cap: implied that \[ \frac{B_e}{2} = j\cdot0.05 \text{pu} \Rightarrow 2X_c = -j\cdot20 \text{pu} \]

\[ Z_{sc} = 0.08+j\cdot2 \text{pu} \]
\[ Y_{sc} = \frac{1}{Z_{sc}} \]

\[ 4.646^\circ \text{pu} \]

Add a load.
What happens if we attach a load at

"LOAD" = 1.0 + j0.5 pu. = \( S \)

\( \text{Bus 2?} \)

\( Q = \frac{V^2}{X} = V^2 \beta \)

If we assume

Load 5 is given

for (\( V = 1.0 \) pu.)

then we can

approximate \( Z_{LOAD}, Y_{LOAD} \)
\[ [Z_{bus}] \text{ vs. } [Y_{bus}] \text{? Which is better?} \]

\[ [Z_{bus}] = [Y_{bus}] \]

- \( Z_{mm} = Z_{TH} \)
- Good for Fault Studies, hand calcs.

\( Y \) is better:
- Sparse, large systems, easy storage.
- Large systems, better throughout...

- 10,000 x 10,000
- 180,000,000
- 800 MB