Topics for Today:

- Announcements
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - ASPEN software - run off of MTU server via internet, see e-mail instructions.
  - Office: EERC 614. Phone: 906.487.2857
  - Recommended problems & all solutions: 13 solns now posted.
  - Homework Syst Op - due this Friday, Dec 4th.

Ongoing topics...

- Chapter 13 - Power system operation
  - Constrained optimization methods - LaGrange multipliers
  - Optimal Dispatch, Generator Scheduling
    - Economics
    - Other constraints - environmental, contractual, availability
    - System load characteristics
  - Application to lossless system
  - System including losses - use [B] loss coefficient matrix
Economic Dispatch - Optimum allocation of generation among system generators.
Goal: Maximize system efficiency
Minimize system losses (can't bill customers)

Specifics:
Control voltage/vars
- Adjust generator exciter
- Reactors, caps (shunt)
- Tap-changing transformers

Control Power Flow
- Control $P_{gen}$ at each generator
- Phase-shifting transformers
- Line switching

Frequency - (later)
- Prime mover control (droop controller)
- Load management

$$P_{GEN} = P_{loads} + P_{TRANS/DIST Losses}$$

$$P_G = \sum_{i=1}^{n} P_{Gi}$$

How should $P_G$ be divvied up among the n units?
Constraints:

- Online time regimes - Coal 8 hrs
  - Some units down for maintenance
  - Should have rolling/spinning reserve in case units fail.

  \[ \text{Thermo constraints} \quad \text{IR of stator of turbine.} \]

**Hinge - Load Characteristics:**

A) Daily -

![Daily Load Graph]

B) Weekly -

![Weekly Load Graph]

C) Annual -

![Annual Load Graph]
Load Duration Curve

Load Factor: \[ LF = \frac{\text{Energy Used}}{\text{Peak Power} \times \text{hrs}} \]

0.4 - Bad
0.85 - Good

Strategy:

Peak firms: hydro, gas turbine, pumped storage
Base Load - Nukes, Coal

Lose Money
Break Even
Make Money
Ways a utility can make money:

- Raise rates  PUC/PPC must approve  
- Sell more base load MWH  
- Reduce peak load (Load management)  
  - Interruptible loads - water heaters, etc  
  - Time of day rates  
- Increase efficiency  
  - Reduce Aux use in plant (10-15%)  
  - Improve thermal efficiency (Net Heat Rate)  
* Economic Dispatch

For each unit:

\[ \text{HR} = \frac{\text{Input Thermal power, BTU/hr}}{\text{Electrical output}} \]

Typical: \(10.5 \times 10^6 \text{ BTU/MWht}\)

Recognize form as \(\overline{Y}n\)
But one BTU/hr = 0.293 W

\[
M = \frac{1}{HR \times 0.293 \times 10^{-6}} = \frac{3.413 \times 10^6}{HR}
\]

Operating cost of unit \(i\)

\[
C_i = F_i \cdot P_i
\]

- Input Power in MBtu (Order of Mag.)
- Fuel cost in $/MBtu (#1.50 + labor, supplies, maint).

Empirically,

\[
C_i = \alpha_i P_i^2 + \beta_i P_i + \gamma_i
\]

\(\alpha, \beta, \gamma\) in $/hr
The curve is empirically described as:

\[ C_i = a_i + b_i x_i + c_i x_i^2 + d_i x_i^3 + e_i x_i^4 \]

Again, \( P_i = P_L + P_{RL} \)

Problem: solve for \( n \) \( P_{gi} \)'s subject to constraints.

Simplest mathematical formulation is to use Lagrange Multipliers.

**Objective function:**

\[ \text{Min} \quad F(x_1, x_2, x_3 \ldots x_n) \]

**Constraints:**

\[ G_1(x_1, x_2 \ldots x_n) = 0 \]

\[ \vdots \]

\[ G_m(x_1, x_2 \ldots x_n) = 0 \]

Usually (for our purposes) \( m = 1 \)

1. Form the Lagrangian:

\[ L = F(x_1, x_2, \ldots x_n) - \lambda \sum G(x_1, x_2, \ldots x_n) \]

2. Find all partial derivatives of \( L \) wrt \( x_1, x_2 \ldots x_n \) and set them equal to zero.
3) Solve for \( (x_1, x_2, \ldots, x_n), \) \( A \)
from partial derivatives \& \( G(x_1, x_2, \ldots, x_n) \)

4) Establish whether solution is min/max or saddle point. (Evaluate Hessian Matrix)
   - Min if positif definite
   - Local Max if neg def, Saddle if indef.

**Ex: 81**

Box of dimensions \( x, y, z \)

Maximize volume for \( S = 432 \text{ cm}^2 \)

**Objective function:** \( V = xyz \)

**Constraint:**

\( 2(x y + 2x^2 + 2xy) - 432 = 0 \)

\( \frac{dF}{dx} = 2x^2y - 2(4x^2 + 6xy - 432) \)

\( \frac{dF}{dy} = 4x - 87x + 6y = 0 \)

\( \frac{dF}{dz} = 2x^2 - 62x = 0 \)

Constraint= \( 2(x y + 2x^2 + 2xy) - 432 = 0 \)

Solve simultaneously:

\( a = 2 \)
\( x = 6 \text{ cm} \)
\( y = 8 \text{ cm} \)
\( v = 576 \text{ cm}^3 \)
Applying to Economic Dispatch:

Objective: \[ C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \]

Constraints: \[ G = P_g - P_L = 0 = \sum_{i=1}^{n} P_{gi} - \sum_{i=1}^{n} P_{gi}^{\text{loads}} \]
(ignoring line losses for now) \( \text{gen} \), \( \text{loads} \).

1. \[ \lambda = C - \sum_{i=1}^{n} \left( \alpha_i P_{gi} - P_{gi} \right) \]

2. Partial Derivatives:
\[ \frac{\Delta C}{\Delta P_{gi}} = \frac{\Delta C}{\Delta P_{gi}} - \sum_{i=1}^{n} \left( \lambda \right) = \frac{\Delta C}{\Delta P_{gi}} - \lambda \]

Since \( \lambda \) is the same in every term, one way to satisfy conditions is:
\[ \frac{\Delta C}{\Delta P_{gi}} - \lambda = 0, \quad \frac{\Delta C}{\Delta P_{gi}} - \lambda = 0 \quad \ldots \quad \frac{\Delta C}{\Delta P_{gi}} - \lambda = 0 \]

Therefore, each plant must be at same incremental cost, \( \lambda_i \) (\( \lambda_1 = \lambda_2 = \ldots = \lambda_n = \lambda \))

For each unit,
\[ \lambda_i = \frac{\Delta C_i}{\Delta P_{gi}} = 2\alpha_i P_{gi} + \beta_i \]
Example 8.2

Unit 1: $25\text{MW} \leq P_{g1} \leq 150\text{MW}$

$$C_1 = 0.01 \frac{P_{g1}^2}{\text{MW}} + 2 \frac{P_{g1}}{\text{MW}} + 100$$

Unit 2: $30\text{MW} \leq P_{g2} \leq 200\text{MW}$

$$C_2 = 0.004 \frac{P_{g2}^2}{\text{MW}} + 2.6 \frac{P_{g2}}{\text{MW}} + 80$$

How to divide $P_{g1} \# P_{g2}$ within range $55\text{MW} \leq P_L \leq 350\text{MW}$?

For ex., $P_L = 282\text{MW}$

First, select $S_{base} = 100\text{MVA}$ & convert data to p.u.

\(x_1 = (100^2)(0.01) = 100\) \(x_2 = 40\)
\(\beta_1 = (100)(2) = 200\) \(\beta_2 = 260\)
\(\gamma_1 = 100\) \(\gamma_2 = 80\)

\[0.25 \leq P_{g1} \leq 1.50\text{ p.u.}\]
\[0.30 \leq P_{g2} \leq 2.00\text{ p.u.}\]
\[0.55 \leq P_L \leq 3.50\text{ p.u.}\]

\[
\lambda_1 = \frac{\partial C_1}{\partial P_{g1}} = 200 P_{g1} + 200
\]
\[
\lambda_2 = \frac{\partial C_2}{\partial P_{g2}} = 80 P_{g2} + 260
\]

$P_{g1} + P_{g2} = 282\text{ p.u.}$
Solving, setting $\lambda_1 = \lambda_2 = \lambda$

$P_{G1} = 102 \text{ p.u.} \quad (102 \text{ MW})$

$P_{G2} = 180 \text{ p.u.} \quad (180 \text{ MW})$

Looking at complete range, $55 \text{ MW} \leq P_c \leq 355 \text{ MW}$

@ .55 p.u.  $P_{G1} = 0.25 \ , \ P_{G2} = 0.30$

$\lambda_1 = 250 \ , \ \lambda_2 = 284$

$\rightarrow$ must increase unit 1 first, until $\lambda_1 = 284$. This happens at $P_{G1} = \frac{284 - 200}{200} = 0.42 \text{ p.u.}$

Then $\lambda_1$ & $\lambda_2$ can be equal until one unit hits $P_{\text{max}}$. @ .35 p.u., $\lambda_1 = 500 @ P_{G1} = 1.5 \text{ p.u.}$, $\lambda_2 = 420 @ P_{G2} = 2.0 \text{ p.u.}$

$P_{G2}$ limits out first.  $P_{G1} = \frac{420 - 200}{200} = 1.1 \text{ p.u.}$

$\rightarrow$ From $P_c = 3.10$ and up, only $P_{G1}$ increases.
8.1 The Economic Dispatch Problem

We first select \( S_{\text{base}} = 100 \text{ MVA} \); then convert all data into per-unit.

\[ \alpha_1 = (S_{\text{base}})^2 \cdot 0.01 = 100 \quad \alpha_2 = 40 \]

\[ \beta_1 = (S_{\text{base}})^2 \cdot 2.00 = 200 \quad \beta_2 = 260 \]

\[ \gamma_1 = 100 \quad \gamma_2 = 80 \]

\[ 0.25 \leq P_{g_1} \leq 1.50 \text{ pu} \]

\[ 0.30 \leq P_{g_2} \leq 2.00 \text{ pu} \]

\[ 0.55 \leq P_L \leq 3.50 \text{ pu} \]

\[ \lambda_1 = \frac{\partial C_1}{\partial P} = 200P_{g_1} + 200 \]

\[ \lambda_2 = \frac{\partial C_2}{\partial P_{g_1}} = 80P_{g_2} + 260 \]

Let us tabulate and plot results as we develop them, as shown in Figure 8.3. We start by calculating \( \lambda_1 \) and \( \lambda_2 \) for minimum-generation conditions (point 1). Observe that \( \lambda_2 > \lambda_1 \). Since we wish to make the \( \lambda \)'s equal, the strategy is to load unit 1 first. We do this until \( \lambda_1 = 284 \), which occurs at

\[ P_{g_1} = \frac{284 - 200}{200} = 0.42 \text{ (point 2)} \]

Now, calculate \( \lambda_1 \) and \( \lambda_2 \) at the maximum-generation condition (point 3). Observe that \( \lambda_1 = \lambda_2 \).

\[ \lambda_1 = 284 \]

\[ \lambda_2 = 284 \]

Results are presented in Table 8.1.

### Table 8.1. Results for Example 8.2.

<table>
<thead>
<tr>
<th>Point</th>
<th>( P_{g_1} )</th>
<th>( P_{g_2} )</th>
<th>( P_L )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.30</td>
<td>0.55</td>
<td>250</td>
<td>284</td>
</tr>
<tr>
<td>2</td>
<td>0.42</td>
<td>0.30</td>
<td>0.72</td>
<td>284</td>
<td>284</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>2.00</td>
<td>3.50</td>
<td>500</td>
<td>420</td>
</tr>
<tr>
<td>4</td>
<td>1.10</td>
<td>2.00</td>
<td>3.10</td>
<td>420</td>
<td>420</td>
</tr>
</tbody>
</table>

In the case where the incremental cost functions \( \lambda_i \) are linearized, a simple straightforward general solution is possible

\[
2\alpha_i P_{g_i} - \lambda = -\beta_i \quad i = 1, 2, \ldots, n \tag{8.13a}
\]

\[
P_{g_1} + P_{g_2} + \cdots + P_{g_n} = P_L \quad \tag{8.13b}
\]

\[
\begin{bmatrix}
2\alpha_1 & 0 & 0 & \cdots & 1 & P_{g_1} & -\beta_1 \\
0 & 2\alpha_2 & 0 & \cdots & 1 & P_{g_2} & -\beta_2 \\
0 & 0 & 2\alpha_3 & \cdots & 1 & P_{g_3} & \cdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
1 & 1 & 1 & \cdots & 0 & -\lambda & P_L
\end{bmatrix}
\]

\[
\begin{bmatrix}
P_{g_1} \\
P_{g_2} \\
P_{g_3} \\
\vdots \\
P_{g_n} \\
-P_L
\end{bmatrix}
\]

Solve the linear set for the \( P_{g_i} \)'s and \( \lambda \).