EE 5200 - Lecture 24
Fri Oct 24, 2014

Topics for Today:

• Announcements
  • Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  • Office hrs: 4-5pm M,W; 10-11 Friday
  • Office: EERC 614. Phone: 906.487.2857
  • Book exercises from Ch.6,7 solutions posted
  • Next homework: Matlab.

Chapter 7 - Network Equations, Admittance Approaches

• How’s your linear algebra? Time to make use of it...
• Basic strategy for building up \([Y]\) for whole network
• Quick recap of xfmrs and lines.
• Generators
• Example of building \([Y]\) for 4-bus system.
• Network Reduction (Kron Reduction)
• Solution of matrix equations (system of linear equations)
• Upcoming homework - intro to Matlab, matrices, equations.
7.1 BRANCH AND NODE ADMITTANCES

FIGURE 7.3
Single-line diagram of the four-bus system of Example 7.1. Reference node is not shown.

\[
\begin{bmatrix}
\times \quad \times \\ \\
\times \quad \times
\end{bmatrix}
\]

FIGURE 7.4
Reactance diagram for Fig. 7.3. Node (0) is reference, reactances and voltages are in per unit.

\[y_{34} = y_{43} = 0\]

"topology"

admittance matrix for each of the network branches and then write the nodal
\begin{align*}
I_n &= \frac{V_{th}}{\sum_{i=1}^{m} Y_i} \\
I_n &= V_{th} Y_n
\end{align*}

\[ [Z] = [Y] \]

The order in which the labels are assigned is not important here, provided columns and rows follow the same order. However, for consistency with sections let us assign the node numbers in the directions of the branch current Fig. 7.5, which also shows the numerical values of the admittances. Combining these elements of the above matrices having identical row and column labels gives:

\[
\begin{bmatrix}
1 & 2 & 3 & 4 \\
1 & (Y_e + Y_g + Y_f) & -Y_e & Y_e & Y_f \\
2 & -Y_e & (Y_e + Y_g + Y_f) & -Y_e & Y_f \\
3 & -Y_e & -Y_e & (Y_e + Y_g + Y_f) & 0 \\
4 & -Y_f & -Y_e & 0 & (Y_e + Y_f + Y_g)
\end{bmatrix}
\]
Easy to use for S.C., or other.

S.S. Go-H.E. (or 50Hz) calcs.

- Hard to construct/modify.

Easy to call or modify in its use for S.C. studies.

Ex: Fig. 7.5 in text

\[
\begin{bmatrix}
-14.5 \\
8 \\
4 \\
-17 \\
4 \\
4.5 \\
5 \\
0 \\
83
\end{bmatrix}
\]

\[
[\mathbf{Z}_{\text{bus}}]
\]

[\mathbf{Y}_{\text{bus}}]

Checks w/ p. 245
Modification is easy:

ex: Remove Line 2-3: (\text{-}j4)

\[
\begin{bmatrix}
-14.5 & 8 & 4 & 2.5 \\
8 & -13 & 0 & 5 \\
4 & 0 & -4.810 \\
2.5 & -5 & 0 & -8.3
\end{bmatrix}
\]

[y_{\text{bus}}] is SPARSE in general.

Typical "grid" system being analyzed will have 100's or 1000's of buses.

\begin{align*}
\{y_{12} = 0, \quad \text{no line or xfar from 1-2.} \\
y_{21} = 0
\end{align*}
Typically, only 2-5 buses are connected to a given. Most off-diagonal entries of $[Y]$ are 0. When many entries of a matrix are 0, it's a **sparse** matrix.

- Don't have to store zero values.
  - Single-precision complex values:
    - 8 bytes.

For 10,000 bus system:

$\Rightarrow 800$ MB of RAM.

Use linked-link storage, only store the non-zero values.
If each bus is connected to 4 others, 5 then each row has 5 entries.  

\[ \rightarrow 50,000 \text{ non-zero entries} \]

\[ \text{only } 400 \text{ KB needed.} \]

Import: \([Z] = [Y]^T\) is a full matrix.

Must use factorization methods to obtain desired entries in \([Z] \Rightarrow \text{can find} \]

\[ Z = \begin{bmatrix} \end{bmatrix} \]
1. How about its off-nominal turns ratio Xfmr 2?

2. How to modify Xfmr [5]?
No phase shift.

\[
\begin{bmatrix}
y & -y \\
-y & y \\
\end{bmatrix}
\]

\[
y_{55} = y_{55} + y \\
y_{71} = y_{71} + y \\
y_{57} = y_{57} - y \\
y_{75} = y_{75} - y \\
\]

\[
y = \frac{1}{\sqrt{R + jx}}
\]
\[ y_{55} = y_{55}^+ + y_{55}^- + j \frac{B_c}{2} \]

\[ y_{57} = y_{57}^+ - y_{55}^- \]

\[ [y_{55}^- y_{55}] \]

\[ [y_{75}^- y_{77}] \]

\[ \sum \text{Row} 5^- = \frac{\text{Total Shunt}}{\text{Admittance at Bus 5}^-} ? \]
2-winding Xfmr

Y1 Y2 (6° phase shift)

(Bus 5) A

jUL R 1

2sc

7

(A' Bus 7)

[\text{System}]

\begin{bmatrix}
1 & & & & \underline{5} \\
& & & & \underline{5} \\
7 & & & & \\
\end{bmatrix}

\text{10-bus system}

y_{57} + \frac{1}{2sc}

y_{77} + \frac{1}{2sc}
In General:

\[
\begin{bmatrix}
\bar{y}_{11} & \bar{y}_{12} \\
\bar{y}_{21} & \bar{y}_{22}
\end{bmatrix}
\begin{bmatrix}
\vec{v}_1 \\
\vec{v}_2
\end{bmatrix}
= 
\begin{bmatrix}
\vec{I}_1 \\
\vec{I}_2
\end{bmatrix}
\]

\[
\bar{y}_{11} = \frac{\vec{I}_1}{\vec{V}_1}; \quad \bar{y}_{12} = \frac{\vec{I}_1}{\vec{V}_2} \quad \bar{y}_{12} = \frac{\vec{I}_1}{\vec{V}_2}; \quad \vec{v}_1 = 0
\]

etc.
\[ V_{TH} = I_N + 2I_{TH} \]

\[ Y_i = \frac{1}{2I_{TH}} \]

\[ Y_{in} = Y_i \]

\[ Y = \begin{bmatrix} m & n \\ m & n \end{bmatrix} \]
\[ Y_{\text{Load}} = 1.0 - j0.5 \text{ p.u.} \]

How to add effect into \([Y]\)?

\[
[Y] = \begin{bmatrix}
4.596 & 1 - 67.97^\circ \\
4.64 & 1111.8^\circ \\
5.481 & -60.22^\circ 
\end{bmatrix}
\]

**Note:** Since load is connected to Bus 2 (Bus 2 - Gnd) then only \( y_{22} \) is affected.

\( y_{22} \) (new) = \( y_{22} \) (old) + \( Y_{\text{Load}} \)
where\n\[
I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a}
\]
(7.3)

\textbf{FIGURE 7.1}
Circuits illustrating the equivalence of sources when \( I_S = E_S/Z_a \) and \( Y_a = 1/Z_a \).
3-Winding XFMRS

P
S

See section 2.8 in text.

NAME PLATE EX.
(See last page).

H2
X2
H0/X0

EACH LEG OF CORE

Δ

H1
H0/X0

Δ

Δ
Refer to section 2.8 in text...

\[
\begin{align*}
2pz &= 2p + 2s \\
2pt &= 2p + 2t \\
2st &= 2s + 2t
\end{align*}
\]

{\text{From transformer nameplate.}}

\[
\Rightarrow \begin{align*}
2p \\
2s \\
2t
\end{align*}
\]

Fictitious node, \( N \) is 4x4.

Per-Phase "Star" equiv

- All transfer impedances are positive. OK for most S, C, and Load-flow calcs.
- Neg \( Z_s \) can cause trouble in some computer simulations.

When building \( [Y] \) for system, be aware!

- \( Z_s \) is often negative, but \( Z_{ps} = Z_p + Z_s \)
- Node in star equivalent does not physically exist.
Note: Above is for Δ-eqiv.
If using star-eqiv, must also add star-point as a new bus.
T-Lines

Example: Build [L1].

\[ Z_{sc} = \frac{2}{j0.2\text{pu}} \]

\[ \frac{B_e}{2} = j0.05\text{pu} \Rightarrow 2X_c = -j20\text{pu} \]

\[ Z_{sc} = 0.08+j0.2\text{pu} \]

\[ Y_{sc} = \frac{1}{Z_{sc}} \]

\[ 4.64\angle-68.2^\circ\text{pu} \]

Add a load.
What happens if we attach a load of $Z_{load} = 1.0 + j0.5 \Omega$. Then we can express $V = 1.0 \text{ p.u.}$ Load $S$ is given. If we assume $P = 0.5$ we have $Q = V^2 / R$. If we make $Z_{load} = 6.0 \text{ ohms}$, $P = 1.0 \text{ kW}$, $Q = 0.5 \text{ var}$. Then $V = 1.0 \text{ p.u.}$ Load $S$ is given.

\[
\begin{bmatrix}
4.596 - 6.797i \\
4.641 - 6.797i
\end{bmatrix}
\begin{bmatrix}
4.641 - 6.797i \\
4.641 - 6.797i
\end{bmatrix}
\]
$$[Z_{bus}] \text{ vs. } [Y_{bus}]$$

Which is better?

Z is better:

- $Z_{mm} = Z_{th}$
- Good for Fault Studies, hand calcs.

Y is better:

- Sparse, large systems, easy storage.
- Large systems, better throughout...

$10,000 \times 10,000$

$\frac{100,000,000}{800 \text{ MB}}$