Topics for Today:

- Announcements
  - Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  - ASPEN software - arranging to run off of MTU server via internet.
  - Recommended problems & all solutions: Ch. 7 solns posted.

Chapter 7 - Network Equations, Admittance Approaches

- How's your linear algebra? Time to make use of it...
- Basic strategy for building up $[Y]$ for whole network
- Quick recap of xfmrs and lines

Generators

- Example of building $[Y]$ for 4-bus system

Network Reduction (Kron Reduction)

- Solution of matrix equations (system of linear equations)

Upcoming homework - intro to Matlab, matrices, equations.
Close look at $x = \alpha + j\beta$

$$y = \sqrt{2y} = \sqrt{(0.2 + j0.6)^2/\text{mi}} = j7.3 \text{ mS/mi}$$

Very high %?

⇒ may assume lossless?

$$= 0.00034/\text{mi} + j0.00212 \text{ rad/mi}$$

$$= 0.00034 \text{ neper/\text{mi}} + j0.00212 \text{ rad/\text{mi}}$$

Tells us how much attenuation/mi the wave will experience.

For 250 mi: $\text{Attenuation} = (0.00034 \text{ neper/\text{mi}})(250 \text{ mi})$

$= 0.085$ or 8.5%.
$Z_c = \sqrt{\frac{Z_o}{y}} \Rightarrow \text{Real for lossless.}$

$\sim \frac{300 \Omega \ (0 \text{HD})}{(250-400 \Omega)}$

$\sim 70-80 \Omega \ (\text{cables})$

$\frac{1}{Z_c} = \frac{R+jX_L}{jBC}$
Voltage Reflection Coefficient:
\[ \frac{V_R^-}{V_R^+} = \frac{Z_s - Z_c}{Z_s + Z_c} = P_c \quad \frac{Z_s - Z_c}{Z_s + Z_c} = \rho_s \]

Current Reflection Coefficient
\[ \frac{i_R^-}{i_R^+} = -\frac{V_R^-}{V_R^+} = -P_R \]

\[ \begin{align*}
i_R^+ &= \frac{V_R^+}{Z_c} \\
i_R^- &= -\frac{V_R^-}{Z_c}
\end{align*} \]
Basic Idea:

Per-Phase A-N

Gen Norton Admittance

\[
[Y]
\]

Lines, XFMRS, Shunt Reactors, Shunt Capacitors

\[
[Y][V] = [I_{inj}]
\]
\[ \begin{bmatrix} Z_B \end{bmatrix} = \begin{bmatrix} Y_B \end{bmatrix}^{-1} \Rightarrow \]

...still only need modify

\[ Y_{55}, Y_{57}, Y_{75}, Y_{77} \]

...treat as off-nominal turns ratio.
Phase Shift XFMRS (Fig. 2.22)

Read § 2.9!
Building by inspection:

\[
\begin{array}{cccc}
1 & 1 & 2 & 3 \\
1 & -1 & 0 & 0 \\
2 & 1 & -1 & 0 \\
3 & 1 & 1 & -1 \\
4 & 1 & 1 & 1
\end{array}
\]

\[
\begin{array}{cccc}
-1 & -1 & 1 & 0 \\
0 & -1 & 1 & 0 \\
1 & 0 & -1 & 1 \\
0 & 0 & 1 & -1 \\
1 & 1 & 1 & 1
\end{array}
\]

From KCL

\[
\sum I = 0
\]

\[
- \frac{y_{34}}{y_{44}}
\]
\[ \begin{align*}
\underline{y_{33}} &= \underline{y_{33}} + y_{3-4} \\
\underline{y_{43}} &= \underline{y_{43}} + y_{4-3} \\
\underline{y_{34}} &= \underline{y_{34}} - y_{3-4} \\
\underline{y_{43}} &= \underline{y_{43}} - y_{4-3} \\
\end{align*} \]

\[ y_{3-4} = y_{4-3} \quad \text{if bilateral.} \]

**FACTS** -

- Non-bilateral
- \( y_{mn} \) and \( y_{n-m} \)

**EX:***

- **UPFC** - PI Q
- **SVC** - Shunt Q
- P.S. Transformer
\[ \bar{y}_{11} = \frac{\bar{I}_1}{\bar{V}_1} \bigg| \bar{V}_2 = 0 \quad \bar{y}_{12} = \frac{\bar{I}_1}{\bar{V}_2} \bigg| \bar{V}_1 = 0 \]

\[ \bar{y}_{21} = \frac{\bar{I}_2}{\bar{V}_1} \bigg| \bar{V}_2 = 0 \quad \bar{y}_{22} = \frac{\bar{I}_2}{\bar{V}_2} \bigg| \bar{V}_1 = 0 \]

OPEN- and SHORT-CIRCUIT TESTS
Four Cases

\[
\begin{align*}
R & \ x & C : 1 \\
\overline{C} & : R' & J X' \\
\overline{C} : L & R' & J X' \\
L & : C' & R' & J X' \\
\end{align*}
\]

Next:

\[
\begin{bmatrix}
y_0 & y_{12} \\
y_0 & y_{22} \\
y_2 & y_{12} \\
\end{bmatrix}
\]
Basis Approach: Develop $\pi$-Equiv and handle just like T-Line.

One-Line:

per-unit
per-phase

Top-Changers
- LTC's
- Phase-Shift

Michigan Tech  Instructor: Bruce Mork  Phone (906) 487-2857  Email: bamork@mtu.edu
Tap Changing XFMRs - Variations (p.u. representations)

\[ y_{sc} = \frac{1}{R + jX} \]

1. \( y_{sc} \)
2. \((R+jX)\)
3. \(y_{sc}\)
4. \(\frac{1}{|C|} y_{sc}\)

"C" is off-nominal turns ratio. In general, \( C \) is complex.
- \( C \) is real for LTC.
- \( C \) is complex for PS.

If |C| \( \neq 1 \) then magnitude change.
If \( C \) is complex, phase shift.
TAP-CHANGERS

On One-Line Diags:

Conceptually:

In per unit, nominal transformation "disappears"
Generically, we can describe this block as a 2-node [Y]

\[
\begin{bmatrix}
  y_{11} & y_{12} \\
  y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
  \overline{V}_1 \\
  \overline{V}_2
\end{bmatrix} =
\begin{bmatrix}
  \overline{I}_1 \\
  -\overline{I}_2
\end{bmatrix}
\]

where
Strategically using shorts, we can isolate on the values of \([Y]\).

\[
y_{11} = \frac{-I_1}{V_1} \bigg| \bar{V}_2 = 0
\]

\[
y_{22} = -\frac{-I_2}{V_2} \bigg| V_1 = 0
\]

\[
= \frac{1}{Z_{EQ} / |C|} = |C|^2 Y_{EQ}
\]
\[ I_1 = -\frac{cV_2}{2Ea}; \quad I_2 = -\frac{I_1 \times c^*}{\Omega} = -\left[ \frac{c}{2Ea} \right] c^* \]

Note: \[ \frac{I_2}{I_1} = c^* \]

\[ = \frac{|c|}{2Ea} \frac{V_2}{I_1} \]
If we "reverse engineer" our \([Y]\) into an equivalent 2-bus network, then
Observations:

- LTC (TcUL) has a $c$ that is \textit{Real}.

  \hspace{1cm} \text{\textbullet \hspace{1cm} Transfer Admittances}

  \hspace{1cm} $c \cdot Y_{e\text{q}} = c \times Y_{\text{eq}}$

  \hspace{1cm} $\Rightarrow$ \textit{Bilateral}. $(y_{12}=y_{21})$

- Phase-Shifter (PS) has complex $c$.

  \hspace{1cm} \text{\textbullet \hspace{1cm} Transfer admittances}

  \hspace{1cm} $c \cdot Y_{\text{eq}} \neq c \times Y_{\text{eq}}$

  \hspace{1cm} $y_{12} \neq y_{21}$

  \hspace{1cm} Not \textit{Bilateral}. $[Y]$ \textit{not symm. about main diag.}
Transformer LTC's in the CDF File Format

Tap and impedance location specified in first two entries in branch data section.
- entry 1 is bus non-unity tap is connected to
- entry 2 is bus device impedance is connected to

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Complex turns ratio due to phase shifting transformer split to two entries
- entry 15 is transformer final turns ratio
- entry 16 is transformer (phase shifter) final angle

Examples:

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Mutual Inductance

\[ E_i + E_{i2} = V_{ac} \]

\[ I_{i2} + I_{i1} \]

\[ H_{i1} \rightarrow \]

\[ H_{i2} \rightarrow \]

END VIEW
MUTUAL
INDUCTANCE

See also handout on Basic Magnetic Circuits

Fundamental definition of inductance: \( L = \frac{\Phi}{i} = \frac{N \Phi}{i} \)

Self-Inductance
\[ L_{11} = \frac{N_1 \Phi_{11}}{i_1} = \frac{\vec{\Phi}_{11}}{i_1} = \frac{N_1^2}{R} \]

Mutual Inductance
\[ L_{12} = \frac{N_1 \Phi_{12}}{i_2} = \frac{\vec{\Phi}_{12}}{i_2} = \frac{N_1 N_2}{R} \]

Mutual Inductance
\[ L_{21} = \frac{N_2 \Phi_{21}}{i_1} = \frac{\vec{\Phi}_{21}}{i_1} = \frac{N_2 N_1}{R} \]

Self-Inductance
\[ L_{22} = \frac{N_2 \Phi_{22}}{i_2} = \frac{\vec{\Phi}_{22}}{i_2} = \frac{N_2^2}{R} \]
How to Use the Concept of Mutual Inductance

Two-Port Device:

\[
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\]

\[i_1 - \rightarrow + \quad i_2 - \rightarrow + \quad v_1 - \rightarrow + \quad v_2 - \rightarrow +\]

Note: Reference direction of currents is into terminals at (+) side of voltage.

In time domain:

\[
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} = \begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix} \begin{bmatrix}
di_1/dt \\
di_2/dt
\end{bmatrix}
\]

In phasor domain:

\[
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} = \begin{bmatrix}
j\omega L_{11} & j\omega L_{12} \\
j\omega L_{21} & j\omega L_{22}
\end{bmatrix} \begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
\]

Also of note:
In some texts, since $L_{12}$ and $L_{21}$ are mutual inductances, they are called $M_{12}$ and $M_{21}$. Same thing.
\[ Z' = [Y] \]

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3
\end{bmatrix}
= \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

§7.2 of text.
Assume high (R - 0)

\[ \text{multiply both sides by } \]

\[ [2 \times 2] \times \text{high} (R - 0) \]

\[ \text{L.H.S. = L.H.S.} \]

\[ \text{Multiply both sides by} \]

\[ \text{Let} \]

\[ \text{Assume high (R - 0)} \]

\[ \text{Multiply both sides by} \]

\[ \text{Assume high (R - 0)} \]