Topics for Today:

• Announcements
  • Software: online students - apply for ATP/ATPDraw license, verify licensing when you receive it by e-mail, and we will mail you the install CD.
  • ASPEN software - run off of MTU server via internet, see e-mail instructions.
  • Office: EERC 614. Phone: 906.487.2857
  • Recommended problems & all solutions: Ch.13 solns now posted.
  • Homework Syst Op - due this Friday, Dec 7th.

Ongoing topics...
• Chapter 13 - Power system operation
  • Constrained optimization methods - LaGrange multipliers
  • Optimal Dispatch, Generator Scheduling
    • Economics
    • Other constraints - environmental, contractual, availability
    • System load characteristics
  • Application to lossless system
  • System including losses - use [B] loss coefficient matrix
Minimize $f(x) \uparrow p_{11}, p_{12}, \ldots, p_{nm}$ (cost function) (objective function)

Subject to:

$g_1(x) = 0$
$g_2(x) = 0$
\vdots
$g_m(x) = 0$
$g_{n_1}(x) \leq 0$
\vdots
$g_{n_l}(x) \leq 0$
Economic Dispatch - Optimum allocation of generation among system generators.

Goal: Maximize system efficiency
Minimize system losses (can't bill customers)

Specifics:

Control voltage/vars
- Adjust generator exciter
- Reactors, caps (shunt)
- Tap-changing transformers

Control Power Flow
- Control $P_{gen}$ at each generator
- Phase-shifting transformers
- Line switching

Frequency - (later)
- Prime mover control (droop controller)
- Load management

\[ P_{gen} = P_{loads} + P_{trans/dist losses} \]

\[ P_G = \sum_{i=1}^{n} P_{gi} \]

How should $P_G$ be divvied up among the $n$ units?
Constraints:

- On-line time regimes - Coal 8 hrs
- Some units down for maintenance
- Should have rolling/spinning reserve in case units fail.

\[ P_{\text{min}} < P_i < P_{\text{max}} \]

Therm constraints \( I^2R \) of stator of turbine.

Hingo - Load Characteristics:

a) Daily -

b) Weekly -

c) Annual -

Cold Climate

Warm

Heat

AC

Heat

Winter
Load Duration Curve

Load Factor: \[ LF = \frac{\text{Energy Used}}{\text{Peak Power \times hrs}} \]

- 0.4 - Bad
- 0.85 - Good

Strategy:

- Peak: Hydro, Gas Turbine, Pumped Storage
- Base Load: Nukes, Coal

Lose Money → Break Even → Make Money
Ways a utility can make money:

- Raise rates (PUC/FSC must approve)
- Sell more base load MWH
- Reduce peak load (Load management)
  - Interruptible loads - water heaters, etc
  - Time of day rates

- Increase efficiency
  - Reduce Aux use in plant (10-15%)
  - Improve thermal efficiency (Net Heat Rate)
  * Economic Dispatch

For each unit:

\[ HR = \frac{Input \text{ Thermal power, BTU/hr}}{Electrical \text{ output}} \]

Typical: \( 10.5 \times 10^6 \text{ BTU/MW-hr} \)

Recognize form as \( \frac{1}{\eta} \)
But one BTU/hr = 0.293 W

\[ P = \frac{1}{HR \times 0.293 \times 10^{-6}} = \frac{3.413 \times 10^6}{HR} \]

Operating cost of unit i

\[ C_i = F_i \cdot P_i \]

- Input power in MBTU
- Fuel cost in \$/MBTU (Order of Mag.)
- \$1.50 (labor, supplies, maint).

\[ C_i \] \[ \$/hr \]

\[ \alpha_i \cdot P_{G_i}^2 + \beta_i \cdot P_{G_i} + \gamma_i \]

\[ P_{G_i} \] (on system base, p.u.)

Empirically, \[ C_i = \alpha_i \cdot P_{G_i}^2 + \beta_i \cdot P_{G_i} + \gamma_i \]

\( \alpha, \beta, \gamma \) in \$$/hr$$
Again,

\[ P_4 = P_L + P_{TL} \]

Problem: solve for \( n \) \( P_{4i} \)’s subject to constraints.

Simplest mathematical formulation is to use Lagrange Multipliers.

Objective function:

\[ \min \quad F(x_1, x_2, x_3 \ldots x_n) \]

Constraints:

\[ G_i (x_1, x_2 \ldots x_n) = 0 \]

\[ \vdots \]

\[ G_m (x_1, x_2 \ldots x_n) = 0 \]

Usually (for our purposes) \( m = 1 \)

1) Form the Lagrangian:

\[ L = F(x_1, x_2 \ldots x_n) - \lambda G (x_1, x_2 \ldots x_n) \]

2) Find all partial derivatives of \( L \) wrt \( x_1, x_2 \ldots x_n \)

and set them = 0.
3) Solve for \((x_1, x_2, \ldots, x_n), a\) from partial derivatives \(G(x_1, x_2, \ldots, x_n)\)

4) Establish whether solution is min/max or saddle point. (Evaluate Hessian Matrix)
- Min if pos definite
- Local Max if neg det, Saddle if indef.

**Example**

Box of dimensions \(x, y, z\)

Maximize volume for \(S = 432\ cm^2\)

**Objective function:** \(V = xyz\)

**Constraint:**
\[2(xy + 2x^2 + 2xy) - 432 = 0\]

\[\frac{\partial^2}{\partial x} = 2x^2y - 2(4x^2 + 6xy - 432)\]
\[\frac{\partial^2}{\partial y} = 4x - 8\lambda x + 6y = 0\]
\[\frac{\partial^2}{\partial z} = 2x^2 - 6\lambda x = 0\]

**Constraint:**
\[2(xy + 2x^2 + 2xy) - 432 = 0\]

Solve simultaneously:
- \(\lambda = 2\)
- \(x = 6\ cm\)
- \(y = 8\ cm\)
- \(V = 576\ cm^3\)
Applying to Economic Dispatch:

Objective: \[ C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma_i \]

Constraints: \[ G = P_g - P_L = 0 = \sum_{i=1}^{n} P_{gi} - \sum_{i=1}^{n} P_{gi} \] (ignore line losses for now) gen totals.

1. \[ \lambda = C - \sum_{i=1}^{n} (\alpha_i P_{gi} - P_L) \]

2. Partial Derivatives:

\[ \frac{\partial \lambda}{\partial P_{gi}} = \frac{\partial C}{\partial P_{gi}} - \sum_{i=1}^{n} \alpha_i = \frac{\partial C}{\partial P_{gi}} - \gamma_i \]

Since \( \gamma_i \) is the same in every term, one way to satisfy conditions is:

\[ \frac{\partial C}{\partial P_{g1}} - \gamma = 0, \quad \frac{\partial C}{\partial P_{g2}} - \gamma = 0 \ldots \quad \frac{\partial C}{\partial P_{gn}} - \gamma = 0 \]

Therefore, each plant must be at same incremental cost, \( \gamma_i \) (\( \gamma_1 = \gamma_2 = \ldots = \gamma_n = \gamma \))

For each unit,

\[ \gamma_i = \frac{\partial C_i}{\partial P_{gi}} = 2\alpha_i P_{gi} + \beta_i \]
Example 8.2

Unit 1: \[ 25 \text{ MW} \leq P_{g1} \leq 150 \text{ MW} \]
\[ C_1 = 0.01 \frac{P_{g1}^2}{\text{ MW}} + 2 \frac{P_{g1}}{\text{ MW}} + 100 \]

Unit 2: \[ 30 \text{ MW} \leq P_{g2} \leq 200 \text{ MW} \]
\[ C_2 = 0.004 \frac{P_{g2}^2}{\text{ MW}} + 2.6 \frac{P_{g2}}{\text{ MW}} + 80 \]

How to divide \( P_{g1} = P_{g2} \) within range \( 55 \text{ MW} \leq P_L \leq 350 \text{ MW} \)?

For ex., \( P_L = 282 \text{ MW} \)

First, select \( S_{base} = 100 \text{ MVA} \) & convert data to p.u.

\[
\alpha_1 = (100)(0.01) = 1 \quad \alpha_2 = 40 \\
\beta_1 = (100)(2) = 200 \\ 
\beta_2 = 260 \\
\gamma_1 = 100 \\ 
\gamma_2 = 80
\]

\[ 0.25 \leq P_{g1} \leq 1.50 \text{ p.u.} \]
\[ 0.30 \leq P_{g2} \leq 2.00 \text{ p.u.} \]
\[ 0.55 \leq P_L \leq 3.50 \text{ p.u.} \]

\[
\lambda_1 = \frac{\partial C_1}{\partial P_{g1}} = 200 P_{g1} + 200 \\
\lambda_2 = \frac{\partial C_2}{\partial P_{g2}} = 80 P_{g2} + 260
\]

\[ P_{g1} + P_{g2} = 282 \text{ p.u.} \]
Solving, setting \( \lambda_1 = \lambda_2 = \lambda \)

\[
\begin{align*}
P_{G1} &= 1.02 \text{ p.u.} \quad (102 \text{ MW}) \\
P_{G2} &= 0.80 \text{ p.u.} \quad (180 \text{ MW})
\end{align*}
\]

Looking at complete range, \( 55 \text{ MW} \leq P_L \leq 355 \text{ MW} \)

\[ P_{G1} \]

\[ P_{G2} \]

\[ P_L \text{ p.u.} \]

@ 0.55 p.u. \( P_{G1} = 0.25 \), \( P_{G2} = 0.30 \)

\( \lambda_1 = 250 \), \( \lambda_2 = 284 \)

\[ \lambda_1 = 284. \text{ This happens at } P_{G1} = \frac{284 - 200}{200} = 0.42 \text{ p.u.} \]

Then \( \lambda_1 \) & \( \lambda_2 \) can be equal until one unit hits \( P_{\text{max}} \). @ 3.5 p.u., \( \lambda_1 = 500 \) @ \( P_{G1} = 1.5 \text{ p.u.} \)

\( \lambda_2 = 420 \) @ \( P_{G2} = 2.0 \text{ p.u.} \)

\( P_{G2} \) limits out first. \( P_{G1} = \frac{420 - 200}{200} = 1.1 \text{ p.u.} \)

From \( P_L = 3.10 \) and up, only \( P_{G1} \) increases.
now that \( \lambda_1 > \lambda_2 \), suggesting that we unload unit 1 first until we bring \( \lambda_1 \) down to \( \lambda_2 = 420 \). This happens at

\[
P_{g_1} = \frac{420 - 200}{200} = 1.10 \text{ (point 4)}
\]

Observe that for \( 0.72 \leq P_L \leq 3.10 \), it is possible to maintain equal \( \lambda \)'s. The equations are

\[
\lambda_1 = \lambda_2
\]

\[
200P_{g_1} + 200 = 80P_{g_2} + 260
\]

and

\[
P_{g_1} + P_{g_2} = P_L
\]

These linear relations are plotted in Figure 8.3. For \( P_L = 282 \text{ MW} = 2.82 \text{ pu} \),

\[
P_{g_2} = 2.82 - P_{g_1}
\]

\[
P_{g_1} = 0.4P_{g_2} + 0.3
\]

\[
= 1.128 - 0.40P_{g_1} + 0.3
\]

\[
1.4P_{g_1} = 1.428; \quad P_{g_1} = 1.02 \quad (102 \text{ MW})
\]

\[
P_{g_2} = 2.82 - 1.02 = 1.80 \quad (180 \text{ MW})
\]

Results are presented in Table 8.1.

**Table 8.1. Results for Example 8.2.**

<table>
<thead>
<tr>
<th>Point</th>
<th>( P_{g_1} )</th>
<th>( P_{g_2} )</th>
<th>( P_L )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.30</td>
<td>0.55</td>
<td>250</td>
<td>284</td>
</tr>
<tr>
<td>2</td>
<td>0.42</td>
<td>0.30</td>
<td>0.72</td>
<td>284</td>
<td>284</td>
</tr>
<tr>
<td>3</td>
<td>1.50</td>
<td>2.00</td>
<td>3.50</td>
<td>500</td>
<td>420</td>
</tr>
<tr>
<td>4</td>
<td>1.10</td>
<td>2.00</td>
<td>3.10</td>
<td>420</td>
<td>420</td>
</tr>
</tbody>
</table>

In the case where the incremental cost functions \( \lambda_i \) are linearized, a simple straightforward general solution is possible

\[
2x_i P_{g_i} - \lambda = -\beta_i \quad i = 1, 2, \ldots, n
\]

\[
P_{g_1} + P_{g_2} + \cdots + P_{g_n} = P_L
\]

\[
\begin{bmatrix}
 2x_1 & 0 & 0 & \cdots & 1 & P_{g_1} & -\beta_1 \\
 0 & 2x_2 & 0 & \cdots & 1 & P_{g_2} & -\beta_2 \\
 0 & 0 & 2x_3 & \cdots & 1 & P_{g_3} & \vdots \\
 \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 1 & 1 & 1 & \cdots & 0 & -\lambda & P_L
\end{bmatrix}
\]

(8.13a)

(8.13b)

(8.13c)

Solve the linear set for the \( P_{g_i} \)'s and \( \lambda \).