Topics for Today:

- Course Info:
  - Web page: [http://www.ee.mtu.edu/faculty/bamork/ee5220/](http://www.ee.mtu.edu/faculty/bamork/ee5220/)
  - Book, references, syllabus, more are on web page.
  - Software - Matlab. ATP/EMTP [ License - [www.emtp.org](http://www.emtp.org) ] ATP tutorials posted on our course web page
  - EE5220-L@mtu.edu (participation = half letter grade, 5%)

- HW#8 - Probs. 9.6, 9.12 now past due.
- HW#9 - Probs. 9.2, 9.3, 9.4 due Mon Mar 26, 5pm.
- "Mid-term": tentatively scheduled week of Apr 2-6th.
- Transformer modeling - Section 11.1 of text, plus lecture notes
  - Nonlinear inductor models - Types 93, 98, 96
  - Magnetic materials: B-H characteristics
  - Transformer Inrush - initial conditions
    - Energization inrush
    - Recovery inrush
    - Sympathetic inrush
- Next - take stock of available ATP transformer models

\[ P_e = \frac{1}{L^2} \]
93

\[ R_c \]

II

piece-wise linear

III

\[ i \]

Assumed to begin at (0,0)

98

Monotonic

Rc

\[ i \]

Assumed to include core losses, i.e. \( R_c \)

96

\[ R_c \]
Type-96 - Hysteretic

\[ L = \frac{\Phi}{i} \]
\[ \frac{d}{dt} \Phi = N \frac{di}{dt} \]

Area \& Losses

\[ \begin{cases} 
- \text{Hysteresis} \\
- \text{Eddy Current} \\
- \text{Anomalous} \\
- \text{Stray Losses}
\end{cases} \]

\[ R_c = \infty \text{ if all losses in Type-96} \]
- Approx with linear L for local operating point.
- Refactorize if segment change.

Pseudo-nonlinearity
- Size of network 5
  - Big Network: Type-98 BAD

- No. of inductors
  98 - Bad if lots
  93 - Better

- No. of Segments
  Large: 98 - Bad
  93 - Good

Type-93 - More Stable.
Type-98 - Operates one timestep outside of proper segment.
B-H Scaling

\[ B = \text{Mag Flux Density} \quad \frac{\text{Wb}}{\text{m}^2} \text{ or } T \]

\[ = \frac{\Phi}{A} = \frac{\gamma}{AN} \quad \Rightarrow \quad I = BAN \]
$H = \frac{MMF}{l} = \frac{NI}{l} \frac{A}{m}$ or $\frac{A}{m}$

$H = \text{Magnetic Field Intensity}$

$= \text{MMF drop per unit length along mean path}$

$\text{MMF} = J = NI \text{ Ampere-turns (Ampere-seconds)}$
Initialization \((\theta, i)\) point.

\[ L = \frac{\Delta \theta}{\Delta i} = \frac{\text{Flux}}{\text{Curr}} \]

Initial Flux = \((I(0), F(0))\) 
\((0, -0.45)\)
Inrush:

\[ e(t) = V_m \sin(\omega t + \phi) \]

\[ i(t) = I_m \sin(\omega t + \phi) + I(0) + \phi \]

See next page for details of offsets!
If: \( e(t) = V_m \cos(\omega t + \phi) \quad (\phi = -90^\circ \text{ for sin wave}) \)

Then:

\[
\int_0^t e(t) \, dt = \int_0^t V_m \cos(\omega t + \phi) + \mathbf{A}(0) \, dt \\
= \frac{V_m}{\omega} \sin(\omega t + \phi) - \frac{V_m}{\omega} \sin \phi + \mathbf{A}(0) \\
= +\lambda_m \text{ if } \phi = -90^\circ
\]

\( \mathbf{A}(t) = \lambda_m \sin(\omega t + \phi) - \lambda_m \sin \phi + \mathbf{A}(0) \)

"integration offset" residue flux
$e(t)$

To minimize inrush, switch on at $V_p$ or $-V_p$!
(assumes $I(0) = 0$.)
\( \lambda(0) = 0 \); voltage source (red) is Sine wave which turns on at \( t=0 \). Note worst-case integration offset in flux linked (green).
Special case to illustrate how to get rid of integration offset. Energize transformer at plus or minus peak voltage (Cos voltage function) and then the flux linked will have zero offset. (Again, this assumes that residual flux linked $\lambda(0)$ in transformer core is zero. Unfortunately, $\lambda(0)$ cannot be known or exactly
predicted). Cases below go back to worst-case integration offset to illustrate the characteristics of inrush current. Inrush current spikes lag voltage by 90° as would be expected of an inductance \( L_M \). Winding resistance \( R_1 \) provides damping.
Same case as above, inrush current is overplotted with flux linked. See how flux linked begins with full offset, but the offset decays due to the damping effect of \( R_1 \). Rate of decay is not exactly exponential like in a linear R-L circuit, due to nonlinear (saturable) \( L_M \) characteristic. Decay is initially quite rapid while \( L_M \) is in full saturation, but rate of decay is slower as it progresses (less saturation => smaller current spikes => less damping).