Topics for Today:

- Course Info:
  - Web page: [http://www.ee.mtu.edu/faculty/bamork/ee5220/](http://www.ee.mtu.edu/faculty/bamork/ee5220/)
  - Book, references, syllabus, more are on web page.
  - Software - Matlab. ATP/EMTP [ License - [www.emtp.org](http://www.emtp.org) ] ATP tutorials posted on our course web page
  - [EE5220-L@mtu.edu](mailto:EE5220-L@mtu.edu) (participation = half letter grade, 5%)

- HW#8 - Probs. 9.6, 9.12 due Mon Mar 24th 5pm.
- HW#9 - Probs. 9.2, 9.3, 9.4 due date TBA.
- Term Project - Mar 21st - a) complete reference list and b) fully-detailed table of contents according to format given in Term Project Guidelines, e-mail Dr. Mork.
- Transformer modeling - Section 11.1 of text, plus lecture notes
  - Review pre-req matls on mag circuits(as posted under Pre-Req Mat'ls)
    - Ampere’s Law, Lenz’ Law, magnetic circuit parameters
  - Example of single-phase transformer, Excitation
    - Waveforms for voltage, $I_{EX}$, $I_{R}$, $I_{C}$, $\lambda$
- Next - take stock of available ATP transformer models
REVIEW OF MAGNETIC CIRCUITS

As a simple example, an ideal single-winding magnetic circuit will be used. The magnetic core in this case is assumed to have no magnetic saturation, even at high levels of flux density.

\[ \text{MMF} = \mathbf{F} = N I = R \Phi \]

Some of the basic parameters which physically define this circuit are:

- \( A \) = Cross-sectional area of core
- \( N \) = Number of turns of the winding
- \( \lambda \) = Mean (average) path length of core (dashed line)
- \( \mu \) = Magnetic permeability of the core. \( \mu \) depends on the type of core material. \( (\mu = \mu_r \mu_0) \)

Some other important magnetic circuit quantities are defined as follows:

**Reluctance of magnetic core:**

\[ R = \frac{l}{\mu A} \text{ H}^{-1} \]

**Magnetomotive Force:**

\[ \text{MMF} = N I \text{ Amp-Turns} \]

**Magnetic Flux:**

\[ \phi = \frac{\text{MMF}}{R} = \frac{N I}{R} \text{ Webers} \]

(Direction given by "right-hand rule").

**Magnetic Field Intensity:**

\[ H = \frac{\text{MMF}}{\lambda} \text{ Amp-Turns/m} \]

**Magnetic Flux Density**

\[ B = \frac{\phi}{A} \text{ Webers/m}^2 \] or Tesla

\[ B = \mu H \]

\[ \mu = \frac{\Delta B}{\Delta H} \]
Flux Linkage
\[ \lambda = N \phi = N A B \text{ Weber-Turns (or Volt-sec)} \]

Inductance
\[ L = \frac{\lambda}{i} = \frac{N^2}{R} = \frac{N^2 M A}{\lambda} \text{ Wb-Turns or Henries} \]

Induced Voltage
\[ v(t) = \frac{d\lambda}{dt} = N \frac{d\phi}{dt} = L \frac{di}{dt} \text{ Volts} \]
\[ \lambda(t) = \int v(t) \, dt \]

Note that in this case, the induced voltage \( v(t) \) is zero, since the current and flux do not change with time.

USE OF VARIOUS UNITS OF MEASUREMENT

Manufacturer's test reports for various magnetic materials may give parameters in several different units of measurement. The following is a clarification of these different units:

**Flux Density (B)**

Standard Unit: Tesla = Weber/m²
Other Unit: Maxwells (lines/inch²) = Tesla x 64500
Other Unit: Gauss = Tesla x 10⁴

**Field Intensity (H)**

Standard Unit: Ampere-Turns/m or Amps/m or Amps/cm
Other Unit: Oerstads = Ampere-Turns/m x 0.01257
* Note that "Turns" is not really a dimensional unit

**Magnetomotive Force (MMF)**

Standard Unit: Ampere-Turns
Other Unit: Gilberts = Ampere Turns x 0.4π
SATURABLE MAGNETIC CIRCUITS

As an example, we consider a 1s two-winding transformer with a saturable magnetic core.

\[ e(t) = \frac{d}{dt} \Phi(t) \]

If flux leakage, winding resistance, and core losses are included, the following equivalent circuit can be used:

Note: \( \tilde{v}_p \) and \( \tilde{v}_s \) are voltages measured at terminals. \( \tilde{e}_p \) and \( \tilde{e}_s \) are magnetically induced via core.

\( R_1 \) and \( R_2 \) : AC resistance of windings (linear)
\( R_c \) : Resistance representing core losses (nonlinear)
\( L_m \) : Magnetizing inductance of the core (nonlinear)
\( L_{L1} \) & \( L_{L2} \) : Leakage inductance of windings (linear)

Note that core losses consist of eddy current losses and hysteresis losses, which can be frequency dependent and voltage dependent. Therefore, \( R_c \) cannot be represented as a linear resistance.
Note also that the magnetizing inductance $L_m$ is nonlinear due to magnetic saturation and cannot be represented as a linear inductance.

\[ \lambda = \phi N \]

\[ \mu \text{ is not a constant} \]

\[ \frac{N_i}{L} = H \]

\[ L = \frac{\mu_i}{\mu} \]

\[ \frac{i}{(\mu H_x/N)} \]

In other words, $\mu$ is not constant, and $B$ & $H$ are not linearly related. Therefore, behavior of the transformer core depends on the $B$-$H$ relationship at each instant of time. This nonlinearity must be included when doing transient analysis of transformers.

Hysteresis makes the behavior of the core more complicated than the above $B$-$H$ characteristic indicates. The areas of the hysteresis loops shown below (left) are proportional to the energy required for one steady-state cycle of operation. Each loop corresponds to a different level of voltage. If operation is not steady-state (below, right), a subloop or "re-entrant loop" is followed. 

\[ +B_{\text{max}} \]

\[ -B_{\text{max}} \]

\[ +H_{\text{max}} \]

\[ -H_{\text{max}} \]

\[ h \]

\[ B_x \]

\[ H_x \]

\[ H_{\text{max}} \]

Reentrant loop
Leakage inductance arises because not all the flux links the windings via the core. Figure 4-1 shows an example in which some flux has leaked from the iron core and completed the magnetic circuit through the air. Such leakage is associated with both the primary and secondary windings. For convenience of illustration a core-type transformer with windings on separate limbs is shown. The principle, however, applies to any transformer (or inductor for that matter). If the windings are placed one on top of the other, as is more usual, there will still be leakage inductance, but probably to a lesser degree.

The effect of leakage inductance is as though a small part of the total inductance had been detached and placed in series with the winding, as shown schematically in Fig. 4-2, where $L_p'$ and $L_s'$ are the primary winding and secondary winding leakage inductances, respectively. Again, the effect is generally not important except at relatively high frequencies, for then the reactances are high, and being in series, have a marked effect on performance.
\[
\frac{\partial}{\partial t} N = \frac{\partial}{\partial t} \Phi = N \, \frac{d\Phi(t)}{dt}
\]

Lenz's Law

\[ e(t) = L \frac{dI(t)}{dt} \]

\[ \Phi(t) = NI \]

\[ N \frac{d\Phi(t)}{dt} = \frac{N^2}{2} \]
Steady-State Excitation of Single Phase Transformer
Rated voltage applied to primary, Open-circuited secondary