EE 5220 - Lecture 27

Topics for Today:

- **Course Info:**
  - Web page: [http://www.ee.mtu.edu/faculty/bamork/ee5220/](http://www.ee.mtu.edu/faculty/bamork/ee5220/)
  - Book, references, syllabus, more are on web page.
  - Software - Matlab. ATP/EMTP [ License - www.emtp.org ] ATP tutorials posted on our course web page
  - **EE5220-L@mtu.edu** (participation = half letter grade, 5%)

- HW#8 - Probs. 9.6, 9.12 now past due.
- HW#9 - Probs. 9.2, 9.3, 9.4 due Mon Mar 31, 9am.
- **Mid-term: feedback is that it can work (with one exception) for Apr 3-6th.**
- Term Project - past due: a) complete reference list and b) fully-detailed table of contents according to format given in Term Project Guidelines, e-mail Dr. Mork.
- Transformer modeling - Section 11.1 of text, plus lecture notes
  - Nonlinear inductor models - Types 93, 98, 96
  - Magnetic materials: B-H characteristics
  - Transformer Inrush - initial conditions
    - Energization inrush
    - Recovery inrush
    - Sympathetic inrush
- Next - take stock of available ATP transformer models
piece-wise linear

Monotonic

Assumed to begin at (0,0)

Assumed to include core losses, i.e. Rc
Type-96 - Hysteric

$$L = \frac{\Phi}{I}$$
$$A = N \Phi$$

Weakness - Subloop trajectories.

$$e = \frac{d\Phi}{dt} = N \frac{dB}{dt}$$

\[ \text{"Flux-Current Loop"} \]

Area x Losses

\[ \{ \begin{align*}
- & \text{Hysteresis} \\
- & \text{Eddy Current} \\
- & \text{Anomalous} \\
- & \text{Stray Losses}
\end{align*} \]

$$R_c = \infty \text{ if all losses in Type-96}$$

Rc \Rightarrow \text{Type-96}$$
Type-95

"true nonlinear"

piece-wise Linear

Reversible
Single-Valued
Compensation Method
Newton Iter.

Linear Network
Type 98

Pseudo-nonlinear

- Approx with linear L for local operating point.
- Refactorize if segment change.
- Size of network 5
  - Big Network: Type-98 BAD

- No. of inductors
  98 - Bad if lots
  93 - Better

- No. of segments
  Large: 98 - Bad
  93 - Good

Type-93 - More Stable.
Type-98 - Operates one timestep outside of proper segment.
**B-H Scaling**

- $B$: Magnetic Flux Density \( \frac{Wb}{m^2} \) or \( T \)
- $H$: Magnetic Field Intensity
- $J$: Current Density
- $N$: Number of Turns
- $A$: Cross-sectional Area
- $l$: Mean Path Length

\[
B = \frac{\Phi}{A} = \frac{J}{AN} \implies J = BAN
\]
\[ H \rightarrow i \quad ? \]

\[ H = \text{Magnetic Field Intensity} \]

\[ = \text{MMF drop per unit length along mean path} \]

\[ \text{MMF} = \Phi = Ni \quad \text{Ampere-turns} \]

\[ \text{(Ampere-turns)} \]

\[ H = \frac{\text{MMF}}{l} = \frac{Ni}{l} \quad \frac{A\cdot t}{m} \quad \text{or} \quad \frac{A}{m} \]
Initialization \((\hat{\theta}, i)\) point.

\[ L = \frac{\Delta \theta}{\Delta i} = \frac{\text{Flux}}{\text{Curr}} \]

Initial Flux = \((I(0), \text{Flux}(0))\)
\((0, -0.45)\)
Inrush -

\[ e(t) = V_m \sin \omega t + \phi \]

\[ I(t) = I_m \sin(\omega t + \phi) + I(0) \quad \text{int. offset} \]

See next page for details of offsets!
The given equation is:

\[ \theta(t) = \int_{0}^{t} \sin(\omega t + \phi) \, dt + \phi \]

If \( \phi = -90^\circ \), then:

\[ \theta(t) = \int_{0}^{t} \sin(\omega t + \phi) \, dt + \phi = \int_{0}^{t} \sin(\omega t - 90^\circ) \, dt - 90^\circ \]

The integral of \( \sin(\omega t - 90^\circ) \) is:

\[ \int \sin(\omega t - 90^\circ) \, dt = -\cos(\omega t - 90^\circ) \]

Evaluating from 0 to \( t \):

\[ -\cos(\omega t - 90^\circ) \bigg|_{0}^{t} = -\cos(\omega t - 90^\circ) + \cos(90^\circ) \]

Since \( \cos(90^\circ) = 0 \), we get:

\[ -\cos(\omega t - 90^\circ) \]

So the final expression is:

\[ \theta(t) = -\cos(\omega t - 90^\circ) - 90^\circ \]

Therefore, for \( \phi = -90^\circ \):

\[ \theta(t) = -\cos(\omega t - 90^\circ) - 90^\circ \]
To minimize inrush, switch on at \( V_p \) or \(-V_p \)!

(assumes \( \theta(0) = 0 \)).
$\lambda(0) = 0$; voltage source (red) is Sine wave which turns on at $t=0$. Note worst-case integration offset in flux linked (green).
Special case to illustrate how to get rid of integration offset. Energize transformer at plus or minus peak voltage (Cos voltage function) and then the flux linked will have zero offset. (Again, this assumes that residual flux linked $\lambda(0)$ in transformer core is zero. Unfortunately, $\lambda(0)$ cannot be known or exactly
Cases below go back to worst-case integration offset to illustrate the characteristics of inrush current. Inrush current spikes lag voltage by 90° as would be expected of an inductance \( L_M \). Winding resistance \( R_1 \) provides damping.
Same case as above, inrush current is overplotted with flux linked. See how flux linked begins with full offset, but the offset decays due to the damping effect of $R_1$. Rate of decay is not exactly exponential like in a linear R-L circuit, due to nonlinear (saturable) $L_M$ characteristic. Decay is initially quite rapid while $L_M$ is in full saturation, but rate of decay is slower as it progresses (less saturation => smaller current spikes => less damping).