Topics for Today:

- Course Info:
  - Web page: [http://www.ee.mtu.edu/faculty/bamork/ee5220/](http://www.ee.mtu.edu/faculty/bamork/ee5220/)
  - Software - Matlab. ATP/EMTP [ License - [www.emtp.org](http://www.emtp.org) ]
  - ATP tutorials posted on our course web page
  - [EE5220-L@mtu.edu](mailto:EE5220-L@mtu.edu) (participation = half letter grade, 5%)

- HW#5 will be posted. Partnered exercise. Due Mon Feb 22nd 9am.
  - Section 12.4 - detailed derivation for capacitor
  - Prob 5.3 - first do ATP simulation, then Hand Calculations
  - Prob 5.6

- HW#6 - due ~Mon March 1st 9am.
- Term Project - proposed topic(s) by end of this week, via short e-mail.
- Transmission Lines
  - Recap of T-Line equations
  - Meaning and application of T-Line equations
    - Steady-state phasor calculations, ABCD parameters.
    - Traveling wave calculations
    - Propagation constant, Zc, etc.
  - Use of ATPDraw’s Line Constants to obtain parameters, build line models.
Compensation

Shunt

- Voltage Support
- Power Transfer

\[ P_{1-2} = \frac{V_1 V_2}{X_{12}} \sin(\delta_1 - \delta_2) \]

- Stability

21% increase

(.95 \rightarrow 1.05 \text{ pu.V})
Shunt Comp: Reactors closing

Long T-line

Ferranti Rise

Open

Ref: EE5200
Lightning:

order of

500 kHz
(half period) ≈ 1 ms

Switching:

≈ 250 ms
DISTRIBUTED PARAMETER T-LINES

- "LONG LINES" (>250 km @ 60 Hz)
- FOR LIGHTNING, EVEN VERY SHORT LINES ARE MODELED AS DIST-PARAM.

For incremental length:

\[ I_x = I(x) \]
\[ V_s = V(x + \Delta x) \]
\[ V_x = V(x) \]
\[ \Delta x \]

\[ Z = zL = R + jX \]
\[ Y = yL = G + jB \]
Making $\Delta x$ very small, (Small $z$)

\[
\begin{align*}
   dv &= i z \, dx \\
   dI &= v y \, dx
\end{align*}
\]

Rearranging,

\[
\begin{align*}
   \frac{d^2v}{dx^2} &= \frac{dI}{dx} z^2 \\
   \frac{dI}{dx} &= v y
\end{align*}
\]

Taking derivative of (1),

\[
\frac{d^2v}{dx^2} = \frac{dI}{dx} z^2
\]
Substituting into (2) \[ \frac{d^2V}{dx^2} = \sqrt{y^2} \]

This implicit general solution:
\[ V = A_1 e^{\sqrt{y^2}x} + A_2 e^{-\sqrt{y^2}x} \]

Since \[ I = \frac{dV}{dx} \]
\[ I = A_1 \sqrt{\frac{y}{z}} e^{\sqrt{y^2}x} - A_2 \sqrt{\frac{y}{z}} e^{-\sqrt{y^2}x} \]

At \( x = 0 \), \( V = V_R \), \( I = IR \)
\[ V(0) = V_R = A_1 + A_2 \]
\[ I(0) = IR = \sqrt{\frac{y}{z}} A_1 - \sqrt{\frac{y}{z}} A_2 \]
Defining \( z_c = \sqrt{\frac{y}{y}} = \frac{\text{Char Imp}}{\text{Imp}} \)

\[ \gamma = \sqrt{y z_c} = \text{Propagation Const.} \]

\[
\begin{align*}
V_R &= A_1 + A_2 \\
I_R &= A_1 - A_2 \\
\text{ } & \quad \frac{I_R}{Z_c}
\end{align*}
\]

\[ \Rightarrow \quad A_1 = \frac{(V_R + Z_c I_R)}{2} \]

\[ A_2 = \frac{V_R - Z_c I_R}{2} \]
\[ I(x) = \frac{V(x)}{\frac{Z_c}{A}\cosh(3x) + \frac{Z_c}{B}\sinh(3x)} \]

\[ V(x) = V_R \cosh(3x) + \frac{Z_c}{Z_{sc}} \sinh(3x) \]

\[ T_s = T(x) \]

\[ V_s = V(x) \]

\[ I(x) = \frac{V_R + \frac{Z_c}{Z_{sc}}}{2} e^{\frac{x}{3}} - \frac{V_R + \frac{Z_c}{Z_{sc}}}{2} e^{-\frac{3x}{x}} \]
If we match Eqns. with \( T - E \equiv \),

From Eqns.:

\[
\frac{V_s}{\frac{V_r}{\frac{1}{x}}}
\]

[AB] with \( [CD] \),

\[
Z = \frac{Z_1}{Z_2}
\]

\[
Y' = Y' = \frac{x}{Z} = \frac{1}{Z}
\]

In hyperbolic form:

\[
\frac{V_r}{\frac{1}{y}}
\]
per-phase T-Line
Propagation Constant

\[ \gamma = \sqrt{y z} = \alpha + j\beta \]

\( \alpha = \text{attenuation constant} \)
\( \text{(neperes/m)} \)
\( \beta = \text{phase constant} \)
\( \text{(radians/m)} \)

Referring to p. 8, exponential form of \( V(x) \) & \( I(x) \).

\[ V(x) = V^+(x) + V'(x) \]

**INCIDENT:**
\[ \frac{V^+(x)}{2} = \frac{\sqrt{R + j\beta} e^{\gamma x} e^{j\beta x}}{2} \]

**REFLECTED:**
\[ \frac{V^-(x)}{2} = \frac{\sqrt{R - j\beta} e^{-\gamma x} e^{-j\beta x}}{2} \]
Conceptually

\[ \mathcal{V}^+(x) \rightarrow \mathcal{V}^-(x) \]

\[ S \rightarrow R \]

\[ \mathcal{V}(x) = \mathcal{V}^+(x) \]

\[ + \mathcal{V}(x) = \mathcal{V}^+(x) \]

\[ - \mathcal{V}(x) = \mathcal{V}^-(x) \]

\[ x = 0 \]

Same thing w/ I's

\[ I^+(x) = \left( \frac{V_R + Z_c I_R}{2 Z_c} \right) e^{ax} e^{j\beta x} \]

\[ I^-(x) = -\left( \frac{V_R - Z_c I_R}{2 Z_c} \right) e^{ax} e^{-j\beta x} \]
- S1L: Surge Imp. Loading
- \[ I = \frac{2X}{\beta} e^{-j\beta x} \]
- \[ V - I = fV \]

@ 60 Hz: \[ I = 3038 \text{ miliamps} \]
\[ V = 182,300 \text{ mils} \]

See following pages for add’l notes and a numerical example for a 3-phase line.
Look at $y$

$$y = a + jb$$

$\alpha$ = attenuation constant

$\beta$ = phase constant

$$V = \frac{V_R + IR}{2} e^{ax} e^{jbx} + \frac{V_R - IR}{2} e^{-ax} e^{-jbx}$$

$$I = \frac{V_R + IR}{2} e^{ax} e^{jbx} - \frac{V_R - IR}{2} e^{-ax} e^{-jbx}$$

Magnitude of

$e^{ax}$ and $e^{-ax}$ change with distance $x$

which makes magnitude of $V$ and $I$ vary.

$e^{jbx}$ and $e^{-jbx}$ change only in angle as $x$ changes. $|e^{jbx}| = |e^{-jbx}| = 1$

$$V_+ = \frac{V_R + IR}{2} e^{ax} e^{jbx} = \text{incident voltage that strikes receiving end}$$

$$V_- = \frac{V_R - IR}{2} e^{-ax} e^{-jbx} = \text{reflected voltage reflected back from receiving end}$$

If load on line = $Z_c$, then reflected voltage = 0 ($\frac{V_R}{IR} = Z_c$). This is called a flat line or infinite line. Normally this never occurs and it is impractical to attempt to do this.
For power systems, \( Z_c = \sqrt{\frac{L}{g}} = \text{surge impedance} \)

if \( R_{\text{line}} = 0 \), \( Z_c = \sqrt{\frac{L}{C}} \)

if \( R_{\text{load}} = |Z_c| \) then the reactive power supplied/consumed by line = 0

\[
\frac{V^2}{X_L} = I^2 X_L \quad \frac{V}{I} = \sqrt{X_L X_c} = \sqrt{\frac{L}{C}}
\]

This is called surge impedance loading.

Load is purely resistive \( R_L = \sqrt{\frac{V^2}{L}} \)

\[
|I_L| = \frac{V_L}{\sqrt{3}} \left( \frac{1}{\sqrt{\frac{L}{C}}} \right) \text{ Amps}
\]

\[
SIL = \sqrt{3} |V_L| |I_L| = \sqrt{3} |V_L| \left( \frac{|V_L|}{\sqrt{3}} \right) = \left[ \frac{|V_L|^2}{V^2} \right] \text{ MW}
\]

where \( V \) is in kV

Sometimes SIL is given in p.u.

wavelength

\[
\lambda = \frac{2\pi}{\beta} \approx 3000 \text{ miles @ 60 Hz}
\]

\[
v = f \lambda \approx \text{speed of light}
\]

\[
\nu = \frac{\omega}{\beta} = \frac{1}{\sqrt{\frac{L}{C} \frac{M}{E}}}
\]
EX: Gross peak: 100 MW, V = 200 kV L-L, PF = 1

\[ X_L = 0.412 + 0.3286 = 0.7406 \text{ ohm/mi} \]
\[ L = 1.965 \text{ mH/mi} \]
\[ X_C = 0.0946 + 0.0803 = 0.1747 \text{ Mho/mi} \]
\[ C = 0.01518 \text{ uf/mi} \]
\[ R = 0.1454 \text{ ohm/mi} \]
\[ Z = 1.454 + j 0.7406 = 0.7547 \angle 78.9^\circ \text{ ohm/mi} \]
\[ Y = \frac{5.724 \times 10^{-6}}{120^\circ} \text{ mho/mi} = \frac{1}{X_C} \]

\[ X_L = \sqrt{\frac{x^2}{3}} = 0.6235 \angle 84.45^\circ = \frac{0.668}{\alpha, \beta} + j \frac{6205}{\beta, \alpha} \]

\[ Z_C = \sqrt{\frac{Z}{8}} = 3.63 \angle 55^\circ \text{ ohm} \]
\[ V_R = 200/\sqrt{3} = 115.2 \text{ kV} \]
\[ I_R = 100 \times 10^3 / \sqrt{3} (200 k) = 283.6 \text{ kA} \]

1. Incload.
\[ V_R^+ = \frac{V_R + I_R Z_C}{2} = 109.85 \angle 262^\circ \text{ kV} \]

2. Refl.
\[ V_R^- = \frac{V_R - I_R Z_C}{2} = 7.42 \angle 42.57^\circ \text{ kV} \]

1. Incload.
\[ V_S^+ = \frac{V_R + I_R Z_C}{2} e^{i\theta} e^{i\phi} = 116.68 \angle 329^\circ \text{ kV} \]

2. Refl.
\[ V_S^- = \frac{V_R - I_R Z_C}{2} e^{-i\theta} e^{i\phi} = 6.986 \angle 7018^\circ \text{ kV} \]

\[ V_{\text{load}} = V_R^+ + V_R^- = 115.2 \text{ kV} \]

\[ \frac{V_S}{V_S} = V_S^+ + V_S^- = 123.04 \angle 31.5^\circ \text{ kV} \]

Checks ok.
\[ \beta = \text{const} = 1 \times 10^{-6} \text{ rad/mi} \]
\[ \lambda = \frac{2\pi}{\beta} = 3038 \text{ miles} \]
\[ V = F_A = 182,300 \text{ mi/sec} \]
\[ I_3^+ = \frac{V_3^+}{Z_c} = 321.43 \frac{138.43^\circ}{\text{A}} \]
\[ I_5^- = \frac{V_5^-}{Z_c} = -19.24 \frac{112.51^\circ}{\text{A}} \]

\[ I_3 = I_5^+ + I_5^- = 304.2 \left( \frac{40.01^\circ}{\text{A}} \right) \]
\[ MVA = \sqrt{3} V_{ls} I_{ls} = \sqrt{3} \left( \frac{213.04}{\text{A}} \right) (304.2) = 112.25 \text{ MVA} \]
\[ P_3 = 112.25 \cos (40.01^\circ - 31.5^\circ) = 111.26 \text{ MW} \]
\[ Q_3 = -16.61 \text{ MVAR} \]
You may work with one homework partner on this if you wish. Using ATPDraw’s Line Constants interface, you will enter the physical design dimensions of a single-circuit and a double-circuit line and obtain the parameters of the line, and use the Verify function to confirm the 60-Hz sequence impedances and line-charging MVA.

For both cases, use the lumped parameter coupled-pi model, assume earth resistivity is 100 Ohm-meters, and create the model for 60-Hz. Check off all possible output requests – this will create a detailed output of all parameter matrices and line parameters in the *lis file.

Case 1: See attached example 5.10 from Glover & Sarma 2nd Ed.

Case 2a: Use line data from attached Prob. 5.37, also from Glover & Sarma 2nd Ed.
Case 2b: Use line data from attached Prob. 5.38, also from Glover & Sarma 2nd Ed.

For each case:
• Provide notes on how you handle the conductor parameters and in general how you used Line Constants.
• Copy/paste the parameter input screens from Line Constants. Provide annotations.
• Copy/paste the Verify output for steady-state 60Hz
  • Sequence impedances and line charging MVARs. Explain meaning.
• Copy/paste the Linecheck output for steady-state 60 Hz. Explore use of selecting output in different units. Explain meaning.
• Provide a printout of the .lis file’s Line Constants output. Provide annotations of the meaning of each of the parameter matrices.
• From *lis printout:
  - Make note of z & y in ohms and S per mile or per meter.
  - Make note of γ, α, β, Zc, τ.
  - For the single-circuit line, calculate the line’s ABCD parameters.

In the next homework, you can begin using the line model to simulate things like Ferranti Rise, traveling waves, line loading effects, and many other performance scenarios.
Figure 5.31
Three-phase 765-kV line for Example 5.10

Neutrals:
2—Alumoweld 7 no. 8
Radius = 0.489 cm
GMR = 0.0636 cm
Resistance = 1.52 Ω/km

Phase conductors:
4—ACSR 954 kcmil, 54/7
Radius = 1.519 cm
GMR = 1.229 cm
Resistance = 0.0701 Ω/km
Bundle spacing = 45.7 cm
Earth resistivity = 100 Ωm
Frequency = 60 Hz
Voltage = 765 kV

Table 5.6 Output data for Example 5.10

<table>
<thead>
<tr>
<th>Series phase impedance matrix $Z_p$ Eq. 5.7.19 Units: Ohms/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.1181E+00 + j5.532E-01$ $0.1009E+00 + j2.340E-01$ $0.9813E-01 + j1.842E-01$</td>
</tr>
<tr>
<td>$0.1009E+00 + j2.339E-01$ $0.1200E+00 + j5.500E-01$ $0.1009E+00 + j2.339E-01$</td>
</tr>
<tr>
<td>$0.9813E-01 + j1.842E-01$ $0.1009E+00 + j2.340E-01$ $0.1181E+00 + j5.532E-01$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Series sequence impedance matrix $Z_s$ Eq. 5.7.25 Units: Ohms/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.3187E+00 + j9.869E-01$ $0.1264E-00 - j9.112E-03$ $-0.1421E-01 - j6.389E-03$</td>
</tr>
<tr>
<td>$-0.1421E-01 + j6.374E-03$ $0.1875E-01 + j3.347E-01$ $-0.2903E-01 - j1.814E-02$</td>
</tr>
<tr>
<td>$0.1262E-01 - j9.117E-03$ $0.3022E-01 + j1.607E-02$ $0.1875E-01 + j3.347E-01$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shunt phase admittance matrix $Y_p$ Eq. 5.11.16 Units: S/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$+j4.311E-06$ $-j7.666E-07$ $-j2.167E-07$</td>
</tr>
<tr>
<td>$-j7.666E-07$ $+j4.439E-06$ $-j7.666E-07$</td>
</tr>
<tr>
<td>$-j2.167E-07$ $-j7.666E-07$ $+j4.311E-06$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shunt sequence admittance matrix $Y_s$ Eq. 5.11.19 Units: S/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0000E+00 + j3.187E-06$ $-0.1219E+06 + j7.036E-08$ $0.1219E+06 + j7.036E-08$</td>
</tr>
<tr>
<td>$0.1219E+06 - j7.036E-08$ $-0.3901E+13 + j4.937E-06$ $0.3544E+06 + j2.046E-07$</td>
</tr>
<tr>
<td>$-0.1219E+06 + j7.036E-08$ $-0.3544E+06 - j2.046E+07$ $0.3901E+13 + j4.937E-06$</td>
</tr>
</tbody>
</table>

Conductor surface electric field strength Eqs. 5.12.1-5.12.5

$E_{max} = 19.3 \text{kV}_{rms}/\text{cm}$

Lateral profile of ground-level electric field strength Eq. 5.12.6
5.31 Rework Problem 5.30 with one neutral wire located 6 m directly above the center phase conductor. Compare the series sequence impedance matrix with that of Problem 5.30.

5.32 Using the LINE CONSTANTS program, compute the shunt sequence admittance matrix for the line in Problem 5.13. Assume an average line height of 20 m and no neutral wires. Compare the computed positive-sequence shunt admittance with the result calculated in Problem 5.21.

5.33 Rework Problem 5.32 with two neutral wires located 7 m above and 8 m to the left and right of the center bundle.

5.34 Using the LINE CONSTANTS program, compute the conductor surface electric field strength and the ground-level electric field strength profile for the line in Problem 5.33. Assume a 100 m right-of-way width.

5.35 Determine the effect of a 10% decrease as well as a 10% increase in phase spacing on the conductor surface electric field strength and on the ground-level electric field strength profile for the line in Problem 5.34.

5.36 Determine the effect of a 10% decrease as well as a 10% increase in the average line height on the conductor surface electric field strength as well as the ground-level electric field strength profile for the line in Problem 5.34.

5.37 Using the LINE CONSTANTS program, calculate the equivalent series sequence impedance matrix and the equivalent shunt sequence admittance matrix for the double-circuit, three-phase line shown in Figure 5.34 with phase arrangement I.

Figure 5.34

Double-circuit line for Problems 5.37 and 5.38

<table>
<thead>
<tr>
<th>Phase conductors:</th>
<th>Neutrals:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1—ACSR 2515 kcmil, 76/19</td>
<td>2—Alumoweld 7 no. 8</td>
</tr>
<tr>
<td>Radius = 2.388 cm</td>
<td>Radius = 0.489 cm</td>
</tr>
<tr>
<td>GMR = 1.893 cm</td>
<td>GMR = 0.0636 cm</td>
</tr>
<tr>
<td>Resistance = 0.0280 Ω/km</td>
<td>Resistance = 1.52 Ω/km</td>
</tr>
</tbody>
</table>

Earth resistivity = 100 Ωm
Frequency = 60 Hz
Voltage = 345 kV

5.38 Rework Problem 5.37 for phase arrangement II shown in parentheses in Figure 5.34. Compare the computed results of the two phase arrangements.
x.lis file:
in c:\atp\atpdraw /lcc/...*lis

Use "line check" also...

Click on

\underline{ATP} \rightarrow \text{Linecheck}