Topics for Today:

- Any Remaining Startup Questions?
- Recap of Mesh and NODE equations from Lecture 1:
  - Symmetric about main diagonal
  - \([Y_{BUS}]\) is invertible, usually
- More on node equation formulations, sparse storage, etc.
- Possible Solution Methods
  - Brute Force Inversion and pre-multiplication
  - in situ methods:
    - Gauss Elimination
    - Gauss-Jordan Elimination
    - LU Factorization
- Matrix “manipulations”
  - Kron Reduction
  - Augmentation
    - Adding constraints (add’l variables) to system of equations
    - Adding a source, short circuit, ideal transformer, etc.
• Homework #1 -- To get started:
  • 1) Go thru videotaped EE5200 MatLab tutorials, refer to Matlab online help for matrix operations and take basic notes for your future reference.
  • 2) Use MatLab to solve the matrix equations for the mesh and the node problems in Lecture 1.
  • 3) Go thru the terminology listed in assignment, look up in text or other references. Find corresponding MatLab functions or capabilities. Take notes on MatLab syntax and application “in’s and out’s”
  • 4) Find out how to enter a sparse matrix into MatLab and document the procedure. Learn how to view network topology via the Matlab spy function.
• Don’t hesitate to send e-mail our group e-mail forum,
• it can be helpful for everyone to contribute questions and comments.
Implications of symmetry:

i.e. if $y_{nk} = y_{kn}$

Bilateral vs. non-Bilateral

\[
\begin{align*}
\begin{cases}
y_{kn} \neq y_{nk} \\
p.s. xfrmr or dependent source
\end{cases}
\end{align*}
\]

Transfer admittances:

\[ -y_{nk} = \frac{I_{nk}}{V_n} \]

\[ -y_{kn} = \frac{I_{kn}}{V_k} \]
- If symmetric about main diagonal, then might get by with staring only lower half of off-diagonal terms.

}  

- Careful! a) in situ methods will produce “fills” Can’t look statically at storage requirements!  
  b) Produce errors in solution if non-bilateral.

Remaining topics:
- Linked list Storage
- Thev \rightarrow Norton for gen’s & \{Y_{bus}\}
- Augmenting \{Y_{bus}\}
- Partitioning (Kron Reduction)
Ex: Coefficient matrix

Full storage: \[ A = \begin{bmatrix}
3 & 0 & 0 & 2 & 0 \\
0 & 5 & 1 & 0 & 0 \\
0 & 1 & 2 & 7 & 9 \\
2 & 0 & 7 & 1 & 0 \\
0 & 0 & 9 & 0 & 4
\end{bmatrix} \]

Single precision: 8
Real: 4 bytes
Complex: 8 bytes
\[ \Rightarrow \times 8 \times 25 = 200 \text{ bytes} \]

\[ 10,000 - \text{bus} \]
\[ \Rightarrow [Y]: 1,000,000,000 \text{ entries} \]
Linked List: Storage

Actual Values

<table>
<thead>
<tr>
<th>A</th>
<th>ICOL</th>
<th>NEXT</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
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</tr>
<tr>
<td>5</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

NBEG

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

↑ INT

pointers to Start of each row

If NBUS < 32,000, then we can use 2-byte Int,

10 4
52
10

13×8

13×2

166 Bytes

KEY:

Vital to understand data structure.

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Data Type

\[ A \]  
- Single-prec Complex \( A \)
- " " Int. ICOL
- " " Int. NEXT

\[
\begin{array}{c|c}
A & 3 \\
ICOL & 1 \\
NEXT & 2 \\
\end{array}
\]
Ex:

Thevenin equivalent of Gen

\[
\begin{align*}
\tilde{E}_1 & \quad + \quad \frac{1}{j10} \quad \tilde{E}_2 & \quad + \quad \frac{j1}{j2} & \quad \tilde{E}_3 & \quad + \quad \frac{j1}{j2} \\
\end{align*}
\]

Convert to admittances w/ Norton equivs

\[
\begin{align*}
\tilde{I}_1 & \quad + \quad \frac{j3}{j1} & \quad \tilde{I}_2 & \quad + \quad \frac{j2}{j1} & \quad \tilde{I}_3 & \quad + \quad \frac{j4}{j1} \\
\end{align*}
\]
Node 1

\[
\begin{align*}
\vec{I}_1 &= (\vec{V}_1 - \vec{V}_2)j^2 + (\vec{V}_1 - \vec{0})(-j10S) \\
\vec{I}_2 &= (\vec{V}_1 - \vec{V}_2)j^3 + (\vec{V}_2 - \vec{V}_3)j^1 + (\vec{V}_2 - \vec{0})(-j1) \\
&\quad + (\vec{V}_2 - \vec{0})(-j2) \\
\vec{I}_3 &= (\vec{V}_2 - \vec{V}_3)(j^1 - j2) + (\vec{V}_3 - \vec{0})(-j4)
\end{align*}
\]

\[
\begin{bmatrix}
\vec{I}_1 \\
\vec{I}_2 \\
\vec{I}_3
\end{bmatrix} = \begin{bmatrix}
\text{or, build} \\
\text{[y] by} \\
\text{inspection.}
\end{bmatrix}
\begin{bmatrix}
\vec{V}_1 \\
\vec{V}_2 \\
\vec{V}_3
\end{bmatrix}
\]

Continue, see homework.
Admittance Equations

General Form: 

\[ \begin{bmatrix} \text{Bus} \\ \text{Vnode} \end{bmatrix} = \begin{bmatrix} \text{I} \\ \text{INJ} \end{bmatrix} \]

We can add constraints:
- V Source
- Short
- Transformer
- Dependent Sources (Op-Amp)
Kron Reduction - System Reduction - Kron Elimination


Possible to reduce to equiv system of fewer nodes.
Goal: Only buses of interest need be observable.

Constraint: Must retain source nodes (nodes at which current is being injected).

Steps:
1) Reorder system, move buses to kept to top, i.e. 1.....K
   Remaining L.....Z nodes are absorbed into system.
2) Perform Kron Reduction.
\[
\begin{bmatrix}
[K] & [L] \\
[L^T] & [M]
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix}
= 
\begin{bmatrix}
I_A \\
I_X
\end{bmatrix}
\]

\[
Y_{bus} V = I
\]

1. \[I_A = KV_A + LV_B\]

2. \[I_X = LT V_A + MV_B\]

Since \[I_X = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\]
\[ I_A = [K-LM^2|^LT][V_A] \]

Substituting \( V_B \) into Eqn. (1):

Substituting Eqn. (2) for \( I_X = 0 \).

Therefore, the reduced system is thus implied to be \([K-LM^2|^LT]\).

Derivation assumes bilateral system (note \( L_T \)).
Reduced \([Y_{bus}]\) is

\[
\begin{bmatrix}
Y_{bus,\ Reduced} \\
\end{bmatrix} = K - L MT^{-1}L^T
\]

**IMPORTANT OBSERVATION:**
If \(L\) & \(LT\) are off-diagonals, then this egn. only valid for bilateral \underline{system}.