Topics for Today:

- Any Remaining Startup Questions?
- Recap of Mesh and NODE equations from Lecture 1:
  - Symmetric about main diagonal
  - \([Y_{BUS}]\) is invertible, usually
- More on node equation formulations, sparse storage, etc.
- Possible Solution Methods
  - Brute Force Inversion and pre-multiplication
  - in situ methods:
    - Gauss Elimination
    - Gauss-Jordan Elimination
    - LU Factorization
- Matrix “manipulations”
  - Kron Reduction
  - Augmentation
    - Adding constraints (add’l variables) to system of equations
    - Adding a source, short circuit, ideal transformer, etc.
•  **Homework #1 -- To get started:**
  • 1) Go thru videotaped EE5200 MatLab tutorials, refer to Matlab online help for matrix operations and take basic notes for your future reference.
  • 2) Use MatLab to solve the matrix equations for the mesh and the node problems in Lecture 1.
  • 3) Go thru the terminology listed in assignment, look up in text or other references. Find corresponding MatLab functions or capabilities. Take notes on MatLab syntax and application “in’s and out’s”
  • 4) Find out how to enter a sparse matrix into MatLab and document the procedure. Learn how to view network topology via the Matlab spy function.
  • Don’t hesitate to send e-mail our group e-mail forum,
  • it can be helpful for everyone to contribute questions and comments.
Implications of symmetry: if \( y_{nk} = y_{kn} \) ?

Bilateral vs. non-Bilateral

\[
\begin{align*}
\text{if } y_{kn} \neq y_{nk} & \text{ if P.S. xfrm or dependent source} \\
\end{align*}
\]

"Transfer admittances"

\[
\begin{align*}
-y_{nk} &= \frac{I_{nk}}{V_n} \\
-y_{kn} &= \frac{I_{kn}}{V_k}
\end{align*}
\]
- If symmetric about main diagonal, then might get by with storing only lower half of off-diagonal terms.
- Careful, a) in situ methods will produce "fills".
  b) Produce errors in solution if non-bilateral.

Remaining topics:
- Augmenting [Ybus]
- Partitioning (Kron Reduction)
- Norton for gen's $i$ [Ybus]
- Linked list storage
Ex: Coefficient matrix

Full storage: \( A = \begin{bmatrix}
3 & 0 & 0 & 2 & 0 \\
0 & 5 & 1 & 0 & 0 \\
0 & 1 & 2 & 7 & 9 \\
2 & 0 & 7 & 1 & 0 \\
0 & 0 & 9 & 0 & 4
\end{bmatrix} \)

25 numbers.

Single precision: 8
Real: 4 bytes
Complex: 8 bytes \( \Rightarrow \) \( 8 \times 25 = 200 \) bytes

10,000-bus
\( \Rightarrow [Y]: 100,000,000 \) entries
Linked List: Storage

Actual Values

<table>
<thead>
<tr>
<th>A</th>
<th>icol</th>
<th>NEXT</th>
<th>NBEG</th>
</tr>
</thead>
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<td>2</td>
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</tbody>
</table>

Complex

INT

13x8

13x2

If NBUS < 32,000, then we can use 2-byte INT.

Vital to understand data structure.

166 Bytes
Data Type

\[ A \]

- Single-prec Complex \( A \)
- " " Int. ICOL
- " " Int. NEXT

\[
\begin{array}{c|c}
A & 3 \\
ICOL & 1 \\
NEXT & 2 \\
\end{array}
\]
Ex: 

The equiv of Gen

Convert to admittances w/Norton equivs
Node 1

\[ \bar{I}_1 = (\bar{V}_1 - \bar{V}_2)j^3 + (\bar{V}_1 - \bar{O})(-j105) \]
\[ \bar{I}_2 = (\bar{V}_2 - \bar{V}_1)j^3 + (\bar{V}_2 - \bar{V}_3)j^1 + (\bar{V}_2 - \bar{O})(-j1) \]
\[ + (\bar{V}_2 - \bar{V}_4)(-j2) \]
\[ \bar{I}_3 = (\bar{V}_3 - \bar{V}_2)(j - j^2) + (\bar{V}_3 - \bar{O})(-j4) \]

\[ \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix} = \begin{bmatrix} \text{or, build} \\ \text{[Y]} \text{ by} \\ \text{inspection.} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \end{bmatrix} \]

Continue, see homework.
Admittance Equations

General Form:

\[
\begin{bmatrix}
Y_{\text{bus}}
\end{bmatrix}
\begin{bmatrix}
V_{\text{node}}
\end{bmatrix}
=
\begin{bmatrix}
I_{\text{INJ}}
\end{bmatrix}
\]

We can add constraints:
- \text{V source bus-bus}
- \text{Short}
- \text{XFMR}
- \text{Dependent sources (OP-AMP)}
Krolik Reduction - System Reduction
- Krolik Elimination


Possible to reduce to equiv system of fewer nodes.

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Goal: Only buses of interest need be observable.

Constraint: Must retain source nodes (nodes at which current is being injected).

Steps:
1) Reorder system - move buses kept to top, i.e. 1,...,K

Remaining L,...,Z nodes are absorbed into system.

2) Perform Kron Reduction.
\[
\begin{bmatrix}
[K] & [L] \\
[L^T] & [M]
\end{bmatrix}
\begin{bmatrix}
V_A \\
V_B
\end{bmatrix} =
\begin{bmatrix}
I_A \\
I_B
\end{bmatrix}
\]

\text{Y}_{Bus} \quad \text{V} \quad \text{I}

1. \quad I_A = KV_A + LV_B

2. \quad I_B = L^TV_A + MV_B

\text{Since } I_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\(3 - L^T V_A = M V_B \quad \text{From Eqn. (2)}\)

for \( I_x = 0 \).

\(4 - M' L^T V_A = V_B \quad \text{premultiply both sides by } M^{-1}\)

Substituting \( V_B \) into Egn. (1),

\[ I_A = K V_A - L M' L^T V_A \]

\[ [I_A] = [K - L M' L^T] [V_A] \]

The \([Y_{bus}]\) for this reduced system is thus implied to be \([K - L M' L^T]\). Derivation assumes bilateral system (note \( L, L^T \))
Reduced \([Y_{bus}]\) is

\[
\begin{bmatrix}
Y_{bus} \\ Reduced
\end{bmatrix} = K - LM^{-1}LT
\]

IMPORTANT OBSERVATION:
If \(L \& LT\) are off-diagonals,
then this eqn. only valid for bilateral system.