Topics for Today:

- EE 5210 - Power System Protection
- Questions from last lectures?
- Questions/Comments on Assignment #7

Topics for Today:
- Optimal Dispatch in "lossless" system
- Optimal Dispatch in real system, with transmission losses included

Assignment #8
- LaGrangian multiplier
- Optimal dispatch (lossless)
- Optimal dispatch (including line losses)
- Run Aspen tutorial (manuals in lab)
Reordering (Assn. #7):

% Fill BV and Solve for [I] vector of injected currents:
BV=zeros(nbus,1);  %vector of solved/known bus voltages
I=zeros(nbus,1);

for n=1:nbus
    BV(n) = complex(busv(n)*cos(busa(n)*pi/180), busv(n)*sin(busa(n)*pi/180));
end
I = Y*BV;  %Solve for injected current vector

nflops=zeros(6,1);  %create empty column vector for flops count

% COLWMD reordering:
flops(1);  %Reset floating point operations counter
%Reorder and fill new Y and I:
Yl=sparse(nbus,nbus);  %Change to Yl=zeros(nbus,nbus) - slower full matrix
Il=zeros(nbus,1);
Reordl=colmmd(Y);
for n=1:nbus
    Il(n) = I(Reordl(n));
    for m=1:nbus
        Yl(n,m) = Y(Reordl(n),Reordl(m));
    end
end

BV1 = inv(Yl)*Il;

% Un-Order and recover correctly-indexed bus voltages:
BV1 = zeros(nbus,1);
for n=1:nbus
    BV1(Reordl(n)) = BV1(n);
end
nflops(1)=flops;

row

nflops =
10552
10232
10160
10086
9748
10332

rand

» ro

nflops =
10552
10232
10160
10086
9748
10332

» ro

nflops =
10552
10232
10160
10086
9748
10332

» ro

nflops =
10552
10232
10160
10086
11556
10332
Key: think in terms of topology. In compiled program, and for larger systems, reordering can greatly improve execution speed.
Optimization:

Objective: $C = \sum_{i=1}^{n} C_i$

$= \sum_{i=1}^{n} \alpha_i P_{Gi} + \beta P_{Gi} + \gamma \quad \$/hr$

$\frac{\$/hr}{hr \cdot MW}$

$\frac{\$/hr}{hr \cdot MW^2}$

$P_a$

$\alpha_i P_{Gi} + \beta P_{Gi} + \gamma$

$(\text{MVAs})^2 \alpha_i P_{Gi} + (\text{MVAs})^2 \beta P_{Gi} + \gamma$
Cost: \( C = \sum_{i=1}^{n} \alpha_i P_{ai}^2 + \beta_i P_{ai} + 7 \)

Constraints: \( G = P_a - P_L = 0 = \sum_{i=1}^{n} P_{ai} - P_L = 0 \)

Ignore losses:

0. \( L = C - \gamma (\sum_{i=1}^{n} P_{ai} - P_L) \)

2. Partials, in general, for gen "i":
\[
\frac{\partial L}{\partial P_{ai}} = \frac{\partial C}{\partial P_{ai}} - \gamma (1) = \frac{\partial C}{\partial P_{ai}} - \gamma
\]
$J$ is the same for every generator:

\[
\begin{align*}
\frac{\partial C}{\partial P_{ai}} - J &= 0, \\
\frac{\partial C}{\partial P_{a2}} - J &= 0, \\
&\ldots
\end{align*}
\]

$J$ is incremental cost of generation in \$/MW ($J = J_1 = J_2 = J_3 = \ldots = J_n$)

At each unit,

\[
J_i = \frac{\partial C_i}{\partial P_{ai}} = 2\alpha_i P_{ai} + \beta_i
\]
Ex: 2 units, Line Losses Ignored

25 MW < $P_{G1} < 150 MW$

$C_1 = 0.1 P_{G1} + 2 P_{G2} + 100 \$/hr$

30 MW < $P_{G2} < 200 MW$

$C_2 = 0.004 P_{G2}^2 + 2.6 P_{G2} + 80 \$/hr$

Unit 1

Unit 2

Has the schedule $P_{G1} \& P_{G2}$ in range $55 \leq P_L \leq 350 MW$?

Ex: $P_L = 282 MW$?

* See page 4a
Actual operating limits. (Increases with Hz pressure).

$p_{\text{min}}$: must operate at some minimum output to have thermodynamic equilibrium and stable electrical operation. (i.e. electrical, mechanical, thermodynamic equilibrium).

Typical simplified characteristic for load-flow studies.
\[ \alpha_1 = 0.01 \left( \frac{\$/hh}{\text{MW} \cdot \text{hr}} \right) (100)^2 = 100 \]
\[ \beta_1 = 2 \left( \frac{\$/hh}{\text{MW} \cdot \text{hr}} \right) (100) = 200 \]
\[ \gamma_1 = \frac{100}{100} = 1 \]

\[ \alpha_2 = 40 \]
\[ \beta_2 = 260 \]
\[ \gamma_2 = 80 \]

**Constraints:**

\[ P_{G1} + P_{G2} + 282 = 0 \]
\[ 0.25 \leq P_{G1} \leq 1.5 \text{ p.u.} \]
\[ 0.30 \leq P_{G2} \leq 2.0 \text{ p.u.} \]
\[ 0.55 \leq P_L \leq 3.5 \text{ p.u.} \]
\[
\frac{\Delta C_1}{\Delta P_{G_1}} = \frac{3(100P_{G_1^2} + 200P_{G_1} + 100)}{2P_{G_1}} = 7
\]

\[
= \frac{200P_{G_1} + 200}{7} = 7
\]

\[
\frac{\Delta C_2}{\Delta P_{G_2}} = \frac{80P_{G_2} + 260}{P_{G_1} + P_{G_2}} = 7
\]

\[
P_{G_1} = 1.02 \text{ p.u.}
\]

\[
P_{G_2} = 1.80 \text{ p.u.}
\]

\[
\lambda = 261.8
\]
$P_{G_1}$

$G_1$

$G_2$

$P_L = P_{G_1} + P_{G_2}$

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\[ \begin{align*}
@ P_L = 0.55 & \quad \lambda_{G1} = 200(0.25) + 200 = 250 \\
& \quad \lambda_{G2} = 80(0.30) + 260 = 284 \\
\end{align*} \]

IF \( \lambda_{G1} \neq \lambda_{G2} \Rightarrow \text{inequality constraints are "active".} \)

Gradually increase \( P_L \) until all \( \lambda \)'s are equal. Then the \( n+1 \) eqns \( \Rightarrow \) give a valid result.

Cheaper to increase \( P_{G1} \) first.

Start increasing \( P_{G2} \) when \( \lambda = 284 \).
\[ J_1 = 284 = 200P_{g1} + 200 \]

\[ \Rightarrow P_{g1} = \frac{284 - 200}{200} = 0.42 \text{ p.u.} \]

At top end:

\[ J_1 = 200(1.5) + 200 = 500 \]

\[ J_2 = 80(2.0) + 200 = 420 \]

\[ \Rightarrow \text{decrease Gen 1 first, start dropping G2 when} \]

\[ J = 420 = 200P_{g1} + 200 \]

\[ P_{g1} = \frac{220}{200} = 1.1 \text{ p.u.} \]