Topics for Today:

- Any Remaining Startup Questions?
- Recap of Mesh and NODE equations from Lecture 1:
  - Symmetric about main diagonal
  - \([Y_{BUS}]\) is invertible, usually
- More on node equation formulations, sparse storage, etc.
- Possible Solution Methods
  - Brute Force Inversion and pre-multiplication
  - in situ methods:
    - Gauss Elimination
    - Gauss-Jordan Elimination
    - LU Factorization

- MATRIX "Manipulations"
  - Kron Reduction
  - Augmentation Methods
Homework #1 -- To get started:

1) Go thru videotaped EE5200 MatLab tutorials, refer to MatLab online help for matrix operations and take basic notes for your future reference.
2) Use MatLab to solve the matrix equations for the mesh and the node problems in Lecture 1.
3) Go thru the terminology listed in assignment, look up in text or other references. Find corresponding MatLab functions or capabilities. Take notes on MatLab syntax and application “in’s and out’s”
4) Find out how to enter a sparse matrix into MatLab and document the procedure. Learn how to view network topology via the Matlab spy function.
Don’t hesitate to send e-mail to EE5240-L@mtu.edu, it can be helpful for everyone to contribute questions and comments.
Implications of symmetry:

i.e. if $y_{nk} = y_{kn}$?

Bilateral vs. non-Bilateral

\[
\begin{cases}
  y_{kn} \neq y_{nk} & \text{if P.S. xfrmr or dependent source} \\
  \text{other case}
\end{cases}
\]

"Transfer admittances"

\[
-y_{nk} = \frac{I_{nk}}{V_n}
\]

\[
-y_{kn} = \frac{I_{kn}}{V_k}
\]
- If symmetric about main diagonal, then might get by with storing only lower half of off-diagonal terms.}

- Careful! a) in situ methods will produce "fills" Can't look statically at storage requirements!
b) Produce errors in solution if non-bilateral.

Remaining topics:
- Linked list storage
- The \( V \rightarrow \text{Norton for gen's } \& \ [Y_{Bus}] \)
- Augmenting \([Y_{Bus}]\)
- Partitioning (Kron Reduction)
Ex: Coefficient matrix

\[ A = \begin{bmatrix} 3 & 0 & 0 & 2 & 0 \\ 0 & 5 & -1 & 7 & 0 \\ 0 & 0 & 2 & 7 & 0 \\ 0 & 0 & 0 & 2 & 7 \end{bmatrix} \]

Full storage:

- 25 numbers:

Single precision:

\[ \Rightarrow 8 \times 25 = 200 \text{ bytes} \]

Real: 4 bytes

Complex: 8 bytes

\[ \frac{10,000 - \text{bus}}{100,000,000 \text{ entries}} \]

\[ (\mathbf{K}) = 10,000,000 \]
Linked List: Storage

Actual Values

\[ \begin{array}{c|c|c|c}
   &icol & next & nbeg \\
\hline
   1 & 3 & 2 & 1 \\
   2 & 2 & 0 & 2 \\
   3 & 5 & 4 & 3 \\
   4 & 1 & 3 & 4 \\
   5 & 2 & 7 & 5 \\
   6 & 7 & 8 & 6 \\
   7 & 9 & 10 & 7 \\
   8 & 2 & 11 & 8 \\
   9 & 7 & 12 & 9 \\
  10 & 3 & 13 & 10 \\
  11 & 4 & 0 & 11 \\
  12 & 3 & 12 & 12 \\
  13 & 5 & 0 & 13 \\
\end{array} \]

Complex

13x8

INT

13x2

INT

5x2

Vital to understand data structure.

104
52
10

\[ 166 \text{ Bytes} \]

If NBUS < 32,000, then we can use 2-Byte Int.
Data Type

\[ A \]

- Single Precision Complex \( A \)
- \( \ldots \ldots \) Int. \( I_{COL} \)
- \( \ldots \ldots \) Int. \( N_{EXT} \)

\[ A(i) = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} \]
Ex:

Thevenin equivalent of Gen:

\[ \tilde{E}_1, \tilde{E}_2, \tilde{E}_3 \]

Convert to admittances w/Norton equivs:

\[ \tilde{I}_1, \tilde{E}_1, \tilde{E}_2, \tilde{E}_3 \]
Node 1

\[
\mathbf{I}_1 = (\mathbf{V}_1 - \mathbf{V}_2)j^3 + (\mathbf{V}_1 - \mathbf{0})(-j10) \\
\mathbf{I}_2 = (\mathbf{V}_2 - \mathbf{V}_1)j^3 + (\mathbf{V}_2 - \mathbf{V}_3)j1 + (\mathbf{V}_2 - \mathbf{0})(-j1) + (\mathbf{V}_2 - \mathbf{V}_4)(-j2) \\
\mathbf{I}_3 = (\mathbf{V}_3 - \mathbf{V}_2)(j1 - j2) + (\mathbf{V}_3 - \mathbf{0})(-j4)
\]

\[
\begin{bmatrix}
\mathbf{I}_1 \\
\mathbf{I}_2 \\
\mathbf{I}_3
\end{bmatrix} = \begin{bmatrix}
\mathbf{V}_1 \\
\mathbf{V}_2 \\
\mathbf{V}_3
\end{bmatrix}
\]

or, build \([Y]\) by inspection.

Continue, see homework.
Admittance Equations

General Form:

\[
\begin{bmatrix}
Y_{bus}
\end{bmatrix}
\begin{bmatrix}
V_{node}
\end{bmatrix}
=
\begin{bmatrix}
I_{INJ}
\end{bmatrix}
\]

We can add constraints:
- V source Bus-Bus
- Short
- XFMR
- DEPENDENT SOURCES (OP-AMP)
Kron Reduction - System Reduction

- Kron Elimination


Possible to reduce to equiv system of fewer nodes.

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2) Perform Krahn Reduction.

Steps:

1) Reorder system.

Constraint: Must retain source.

Goal: Only buses of interest need be observable.

Remaining L - Z nodes

k top, i.e., T...

Move buses top keep to
\[
\begin{bmatrix}
K & L \\
LT & M
\end{bmatrix}
\begin{bmatrix}
VA \\
VB
\end{bmatrix}
= 
\begin{bmatrix}
IA \\
IX
\end{bmatrix}
\]

\[Y_{bus} \quad V \quad I\]

1. \[I_A = KV_A + LV_B\]
2. \[I_X = LTVA + MV_B\]

Since \[I_X = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}\]
From Eqn. (2) for \( I_x = 0 \),

\[ -L^T V_A = M V_B \]

Premultiply both sides by \( M^{-1} \).

Substituting \( V_B \) into Eqn. (1),

\[ I_A = K V_A - L M^I V_A \]

\[ [I_A] = [K - LM^I L^T] [V_A] \]

The \([Y_{bus}]\) for this reduced system is thus implied to be \([K - LM^I L^T]\).

Derivation assumes bilateral system (note \( L, L^T \))
Reduced \([\tilde{Y}_{\text{bus}}]\) is

\[
[\tilde{Y}_{\text{bus}}] = K - L M L^T
\]

**IMPORTANT OBSERVATION:**

If \(L \neq L^T\) are off-diagonals, then this eqn. only valid for bilateral system.