EE 5240 - Lecture 19

Monday Feb 20, 2006

Topics for Today:
- Homework #6 - Due Wed.
- Convergence considerations, generating fractals, etc.
- Constrained Optimization - Basic Methods
- Optimal Dispatch for simple lossless system

Coming up:
- Optimal dispatch including system losses
- Programming details

- Environmental
- Other Issues
Ex: Box $x = y = z$

Max Volume for $S = 432 \text{cm}^2$

Obj: $V = x y z = 2x^2 y$

(cost function)

Constraint: $2 (xy + 2x^2 + 2xy) = 432 = 0$

$L = 2x^2 y - \left[4x^2 + 6xy - 432\right]$
Form the LaGrangian:

1) \[ L = F(x_1, x_2, \ldots, x_n) - \lambda \left[ G(x_1, x_2, \ldots, x_n) \right] \] (obj.)

2) Find all partial derivatives of \( L \) wrt \( x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots \) and set to zero.

3) Solve for \( (x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots) \)

4) Is soln a min/max/saddle point?
\[ 4xy - 87x + 67y = 0 \]

\[ 2x^2 - 6x + 1 = 0 \]

\[ 2(xy + 2x^2 + 2x^3) - 432 = 0 \]

\[ V = 576 \text{ cm}^3 \]

\[ z = 2 \]

\[ x = 6 \]

\[ y = 8 \]
Key: After solving \( \mathcal{L} \) for \((x_1, x_2, \ldots, x_n)\)
Evaluate Hessian

\[
H = \begin{bmatrix}
\frac{\partial^2 F}{\partial x_1^2} & \frac{\partial^2 F}{\partial x_1 \partial x_2} & \cdots \\
\frac{\partial^2 F}{\partial x_1 \partial x_2} & \frac{\partial^2 F}{\partial x_2^2} & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix}
\]

If \( H \) is...
- pos definite \( \rightarrow \) min
- neg def \( \rightarrow \) local max
- indefinite \( \rightarrow \) saddle pt.

...
Saddle-point: Similar to inflection point but for N-dimensional surface.

Optimization in "N-space" 

\[ N = 1 + n \]

where \( n \) = no. of gens.
SEE "INTRO TO DISPATCH" notes first!

Optimization:

Objective: \[ C = \sum_{i=1}^{n} C_i \]

\[ = \sum_{i=1}^{n} \alpha_i P_{Gi} + \beta P_{Gi} + \gamma \text{ \$/hr} \]

\[ \frac{\$}{hr \cdot HW} \]

\[ \frac{\$}{hr \cdot HW^2} \]

\[ \frac{\$}{hr} \]

\[ = \alpha_i P_{Gi} + \beta P_{Gi} + \gamma \]

\[ \left( MVAe \right)^2 \alpha_i P_{Gi} + \left( MVAe \right) \Delta P_{Gi} + \gamma \]