Topics for Today:

- Homework #7 - Due Wed.
- Convergence considerations, generating fractals, etc.
- Reading: §6.3.1; EE5200 Text - §13.1 thru §13.5
- Optimal Dispatch for simple lossless system (recap)
- Simple example for 2-generator system (recap)
- Incremental cost, $\lambda$ (recap)
- Observations on constrained optimization
- Optimal dispatch including system losses

Coming up:

- Hill-Stevenson optimal power flow method (see attached)
- Programming details
Optimal Dispatch

Before: No losses:

$$P_G = \sum_{i=1}^{\hat{E}} P_{G_i} = P_L$$

With losses:

$$P_G = P_L + P_{TL}$$

Constraint:

$$G = P_G - P_L - P_{TL} = 0$$

Main problem is finding

$$P_{TL} = P_G - P_L$$
From Kron, Kirchmeyer, George:

\[
P_{T_L} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ai} B_{ij} P_{aj}
\]

"B" coefficients

\[
= [P_{a1} \ P_{a2} \ldots \ P_{an}] \begin{bmatrix} B_{11} & \ldots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{n1} & \ldots & B_{nn} \end{bmatrix} \begin{bmatrix} P_{11} \\ \vdots \\ P_{nn} \end{bmatrix}
\]
Ex: Small 2-gen system: \( B = \begin{bmatrix} .002 & -.005 \\ -.005 & .015 \end{bmatrix} \)

(typically \( B_{ij} = B_{ji} \) for off-diagonal terms)

\( P_{G1} = 1.25 \) p.u.
\( P_{G2} = 1.75 \) p.u.  \( \text{Calculate } P_{TL} \)

\[
P_{TL} = \begin{bmatrix} 1.25 & 1.75 \end{bmatrix} \begin{bmatrix} .002 & -.005 \\ -.005 & .015 \end{bmatrix} \begin{bmatrix} 1.25 \\ 1.75 \end{bmatrix} = .0272 \) p.u.

\( N = \frac{2.9788}{3.00} = 99.1\% \)
\[ P_{TL} = [P_g]^T [B] [P_g] \]

Problem is actually to determine what \([B]\) is. Key observation: each utility uses their own method. Most methods are based on examining "sensitivities" of \(P_{TL}\) wrt changes in generation output. In a very general sense, \(\frac{\partial P_{TL}}{\partial P_{ai}}\)
For optimized operation, \( T = T_i \) for all \( T_i \)’s.

From before,
\[
C_i = \alpha_i P_{gi}^2 + \beta_i P_{gi} + \gamma
\]

\[
\frac{\partial C_i}{\partial P_{gi}} = 2 \alpha_i P_{gi} + \beta_i
\]

\[
\frac{\partial P_{TL}}{\partial P_{gi}} = 2 \sum_{j=1}^{n} B_{ij} P_{aj}
\]

\( \Leftarrow \) one eqn for each generator.

\( \Leftarrow \) extra equation(s) for constraint(s)
Settings up Lagrangian

Minimise: \[ C = \sum_{i=1}^{n} C_i \] (objective)

Subject to: \[ P_b - P_L - P_{TL} = 0 \] (constraint)

\[ L = \sum_{i=1}^{n} C_i - \lambda (P_b - P_L - P_{TL}) \]
\[ \sum_{i=1}^{n} P_{ai} = P_{b1} + P_{a2} + \ldots + P_{an} \]

\[ \frac{dL}{dP_{ai}} = \frac{dC_i}{dP_{ai}} - \lambda (1) - \lambda \left( \frac{dP_{TL}}{dP_{ai}} \right) \]

related to a "penalty"
**Constraint Equation:**

\[ \sum_{j=1}^{n} P_{ij} - P_L - \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} B_{ij} P_{aj} = 0 \]

\[ PF_i = \frac{1}{1 - 2 \sum_{j=1}^{n} B_{ij} P_{aj}} = \text{Penalty Factor} \]

\[ j_i = \frac{\partial C_i}{\partial P_{ai}} \left( \frac{PF_i}{1 - \frac{\partial P_L}{\partial P_{ai}}} \right) = \frac{\partial C_i}{\partial P_{ai}} (PF_i) \]
Ways to find \( [B] \):

1) "Hill–Stevenson Method"
   Run load flow repeatedly with small changes in \( P_{ai} \).
   Check and examine sensitivity of all \( S_i \)'s to \( P_{ai} \).

Key: Find all \( \frac{\partial S_k}{\partial P_{ai}} \) \( k = 1, n \)

IEEE 14 Bus case: Take 52 partials.

Only takes 4 simulations: vary each \( P_{ai} \).

4 Gen's
13 Buses where \( S \) fluctuates
Numerically evaluate
\[ \frac{\partial \delta_k}{\partial P_{ai}} = \frac{\Delta \delta_k}{\Delta P_{Gi}} \bigg|_{k=1,n} \]

Starting with base case of known or solved-for \( P_{ai} \)'s & \( \delta_k \)'s.

Change \( P_{Gi} \) by 0.01 p.u. or 0.001 p.u. and rerun load flow and calc for all 13 \( S \)'s
\[ \frac{\Delta \delta_k}{\Delta P_{ai}} = \frac{S_{k,base} - S_{k,\text{new}}}{P_{Gi,base} - P_{Gi,\text{new}}} \]
8.2 Economic Dispatch Considering Losses

In section 8.1, the economic dispatch problem was solved neglecting transmission losses. Experience has shown that in some cases, this approximation produces results that are in serious error. We recall the power balance equation

\[ P_o = P_L + P_{TL} \]  
(8.7a)

The revised equation of constraint, considering losses, is then

\[ G = P_o - P_L - P_{TL} = 0 \]  
(8.14)

Recall that the problem's variables are the \( P_o \)'s. Therefore, it is necessary to formulate the losses using the \( P_o \)'s as variables.

Let us consider the losses of the components of the transmission system, specifically, transformers and lines. Transformers have two types of losses: copper and iron. The iron, or magnetic, losses vary with core flux density, which in turn varies with voltage. Since the transformer voltage varies indirectly, and very little, with load, the variation of core loss with load is quite small and not important. However, the copper loss varies as \( I^2R \); the load current varies directly with power loading; and the series resistance loss varies with the square of this current. Thus, the transformer copper loss should be included in the transmission losses. Likewise, the \( I^2R \) losses in the series element of the transmission line constitute transmission losses. The challenge is to functionally relate these \( I^2R \) losses to the \( P_o \) variables.

Many investigators, including Kron, Kirchmeyer, and George have worked on this problem and proposed a loss equation that formulates the loss as a quadratic function of the \( P_o \) 's:

\[ P_{TL} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_o_i B_{ij} P_o_j \]  
(8.15a)

\[ = P_o \{B\} P_o \]  
(8.15b)

where

\[ P_o = \begin{bmatrix} P_{o_1} \\ P_{o_2} \\ \vdots \\ P_{o_n} \end{bmatrix} \quad \text{vector of generated powers.} \]

\[ \{B\} = \begin{bmatrix} B_{00} \\ B_{01} \\ \vdots \\ B_{nn} \end{bmatrix} \quad \text{array with general entry } B_{ij}. \]

The \( \{B\} \) array is symmetrical, such that \( B_{ij} = B_{ji}. \) Computing the \( B \) values, called the \( B \) constants, can be implemented by several techniques. One approach, called the Hill–Stevenson method, calculates \( B \) constants using partial derivatives, some of which are evaluated numerically using data obtained from a series of load flow runs. A description of the method follows.

Observe that

\[ \frac{\partial P_{TL}}{\partial P_{o_i}} = \frac{\partial}{\partial P_{o_i}} \left( \sum_{i=1}^{n} \sum_{j=1}^{n} P_o_i B_{ij} P_o_j \right) \]  
(8.16a)

\[ = \sum_{j=1}^{n} B_{ij} P_o_j \]  
(8.16b)

Furthermore,

\[ \frac{\partial^2 P_{TL}}{\partial P_{o_i} \partial P_{o_j}} = 2B_{ij} = 2B_{ji} \]  
(8.17a)

But the subscript notation is arbitrary, so that

\[ B_{ij} = \frac{1}{2} \left( \frac{\partial^2 P_{TL}}{\partial P_{o_i} \partial P_{o_j}} \right) \]  
(8.17b)

Consider the general power system as we viewed it for power flow in Figure 7.2. The total power injected into the transmission network is:

\[ P_{TL} = \Re \left( \sum_{i=1}^{n} P_i^f \right) \]  
(8.18a)

If the transmission network were lossless, \( P_{TL} \) would be zero (power in = power out); in general, this summation would equal the total transmission system loss \( P_{TL}. \)

\[ P_{TL} = \Re \left( \sum_{i=1}^{n} P_i^f \left( \sum_{j=1}^{b} P_j^b \right)^* \right) \]  
(8.18b)

\[ = \sum_{i=1}^{b} \sum_{j=1}^{b} V_i^f V_j^b \cos(\delta_i - \delta_j - \gamma_{ij}) \]  
(8.18c)

Now, we compute from equation (8.18b):

\[ \frac{\partial P_{TL}}{\partial P_{o_i}} = \sum_{i=1}^{b} \frac{\partial P_{TL}}{\partial P_{o_i}} \frac{\partial P_{o_i}}{\partial \delta_i} \]  
(8.19)

Now, by the chain rule, consider that

\[ \frac{\partial P_{TL}}{\partial P_{o_i}} = \sum_{i=1}^{n} \frac{\partial P_{TL}}{\partial \delta_i} \frac{\partial \delta_i}{\partial \delta_i} \]  
(8.20)

\[ \text{Here, } b \text{ is the total number of buses, and } n \text{ is the total number of generators. In Chapter 7, } n \text{ was the total number of buses.} \]

\[ \text{\( \uparrow \) The simplification is complicated and relegated to an exercise for the student. See problem 8-7.} \]
The troublesome terms are the $\partial \delta_i / \partial P_{G_i}$ factors. The Hill–Stevenson method provides a clever way of evaluating these partial derivatives numerically. A basic load flow case is established that hopefully approximates the ultimate allocation of generation. One approach is to solve the lossless economic dispatch problem as a first approximation. Then increase all bus loads proportionally by a small amount (say, 10%), and allow generator 1 alone to pick up the increased load. The load flow solves for the phase ($\delta_i$) at all $b$ buses, from which the change in phase ($\Delta \delta_i$) can be calculated. Therefore,

$$\frac{\partial \delta_i}{\partial P_{G_i}} \approx \frac{\Delta \delta_i}{\Delta P_{G_i}} = A_{b1}$$

This partial derivative is reasonably constant over a rather wide range of generation and load values. We repeat the process $n$ times, allowing each generator to act alone to pick up the increase in load, such that

$$\frac{\partial \delta_i}{\partial P_{G_j}} \approx \frac{\Delta \delta_i}{\Delta P_{G_j}} = A_{bj} \quad j = 1, 2, \ldots, n$$

The partial derivatives so approximated are represented as the $A$ constants. Let us now compute a second partial derivative

$$\frac{\partial}{\partial P_{G_i}} \left( \frac{\partial P_{TL}}{\partial P_{G_j}} \right) = \frac{\partial^2 P_{TL}}{\partial P_{G_i} \partial P_{G_j}} = \frac{\partial}{\partial P_{G_i}} \left[ \frac{\partial P_{TL}}{\partial P_{G_j}} \right]$$

$$= \frac{\partial}{\partial P_{G_i}} \left[ \sum_{j=1}^{b} \frac{\partial P_{TL}}{\partial P_{G_j}} \left( \frac{\partial \delta_i}{\partial P_{G_j}} \right) \right]$$

or

$$\frac{\partial^2 P_{TL}}{\partial P_{G_i} \partial P_{G_j}} = \sum_{j=1}^{b} \frac{\partial^2 P_{TL}}{\partial P_{G_j} \partial P_{G_i}} A_{bj} A_{mi}$$

Recall that

$$\frac{\partial P_{TL}}{\partial \delta_i} = 2 \sum_{i=1}^{b} V_i V_i g_{ai} \sin(\delta_i - \delta_k)$$

We wish to compute

$$\frac{\partial}{\partial \delta_i} \left( \frac{\partial P_{TL}}{\partial \delta_k} \right) = \frac{\partial^2 P_{TL}}{\partial \delta_i \partial \delta_k}$$

where $\delta_i$ is one specific $\delta_i$. We determine that

$$\frac{\partial^2 P_{TL}}{\partial \delta_i \partial \delta_k} = 2V_i V_i g_{ai} \cos(\delta_i - \delta_k) \quad m \neq k$$

$$= -2 \sum_{i=1}^{b} V_i V_i g_{ai} \cos(\delta_i - \delta_k) \quad m = k$$

To relate this work to the $B$ constants, recall that

$$B_{ij} = \frac{1}{2} \left( \frac{\partial^2 P_{TL}}{\partial P_{G_i} \partial P_{G_j}} \right)$$

Finally,

$$B_{ij} = \frac{1}{2} \left( \sum_{m=1}^{b} \sum_{n=1}^{b} \frac{\partial^2 P_{TL}}{\partial \delta_m \partial \delta_n} A_{m1} A_{n1} \right)$$

We now return to our main concern, that is, solving the economic dispatch problem considering losses.

$$C = \sum_{i=1}^{n} C_i \quad \text{(objective function)}$$

$$P_0 = P_L - P_{TL} = 0 \quad \text{(equation of constraint)}$$

$$S = \sum_{i=1}^{n} C_i - \lambda \left[ P_0 - P_L - P_{TL} \right] \quad \text{(the Lagrangian)}$$

The general partial derivative with respect to $P_{G_i}$ is

$$\frac{\partial S}{\partial P_{G_i}} = \frac{\partial C_i}{\partial P_{G_i}} - \lambda \left( 1 - \frac{\partial P_{TL}}{\partial P_{G_i}} \right) = 0$$

or, we redefine $\lambda$, such that

$$\lambda = \frac{\partial C_i / \partial P_{G_i}}{1 - (\partial P_{TL} / \partial P_{G_i})} = \frac{1}{\beta_i}$$

where

$$\frac{\partial C_i}{\partial P_{G_i}} = 2\alpha_i P_{G_i} + \beta_i$$

and

$$\frac{\partial P_{TL}}{\partial P_{G_i}} = 2 \sum_{j=1}^{n} B_{ij} P_{G_j}$$
and the equation of constraint requires that

$$\left( \sum_{i=1}^{n} P_{G_i} \right) - P_L - \left( \sum_{i=1}^{n} \sum_{j=1}^{n} F_{G_i} B_{ij} P_{G_j} \right) = 0$$

(8.29)

Thus, the condition for economically optimum operation requires the weighted incremental cost functions for all units to be equal (to each other and \( \lambda \))

$$\lambda_i = \lambda$$

(8.30)

The weighting factor is sometimes called the penalty factor of generator \( i(PF_i) \)

$$PF_i = \frac{1}{1 - \left( \frac{\partial^2 P_T / \partial \delta_i}{\partial^2 P_{G_i}} \right)} = \frac{1}{1 - \sum_{j=1}^{n} B_{ij} P_{G_j}}$$

(8.31)

Example 8.3 is useful for illustrative purposes.

**Example 8.3**

A single-line diagram for the system in example 8.2 is shown in Figure 8.4. A base case load flow study on the system provides the following results:

- \( V_1 = 1/\Omega \)
- \( P_{G_1} = 1.3013 \)
- \( P_{L_1} = 2.1000 \)
- \( V_2 = 1/6.616 \)
- \( P_{G_2} = 1.8200 \)
- \( P_{L_2} = 0.7200 \)
- \( P_{TL} = 0.0313 \)

All values in per-unit

![Figure 8.4. System for example 8.3.](image)

(a) Calculate the \( A \) constants. The load at each bus was increased 10%. If unit \( 1 \) picks up the load,

\[ \Delta \delta_1 = 0 \] (using bus 1 as phase reference)

\[ \Delta \delta_2 = 6.187 - 6.616 = -0.429^\circ \] \((-0.007487 \text{ rad})\)

\[ \Delta P_{G_1} = 1.3094 - 1.0313 = 0.2781 \]

\[ A_{11} = 0 \]

\[ A_{21} = \frac{-0.007487}{0.278100} = -0.026924 \]

(b) Calculate the \( B \) constants


\[ g_{11} = g_{22} = 2.353 \]

\[ g_{12} = g_{21} = -2.353 \]

\[ m = k \]

\[ \frac{1}{2} \left( \frac{\partial^2 P_T / \partial \delta_1}{\partial \delta_2} \right) = -\sum_{i=1}^{n} \sum_{k=1}^{n} V_i V_k g_{ik} \cos(\delta_i - \delta_k) \]

For any \( m \neq k \)

\[ \frac{1}{2} \left( \frac{\partial^2 P_T / \partial \delta_m}{\partial \delta_1} \right) = V_m V_1 g_{m1} \cos(\delta_m - \delta_1) \]

Finally,

\[ B_{ij} = \frac{1}{2} \sum_{m=1}^{n} \sum_{k=1}^{n} \frac{\partial^2 P_T}{\partial \delta_m \partial \delta_k} A_{mi} A_{kj} \]

\[ = 2.337(A_{11}A_{1j} - A_{11}A_{2j} - A_{12}A_{1j} + A_{12}A_{2j}) \]

\[ B_{11} = 2.337[-(-0.026924)^2] = 0.001694 \]

\[ B_{12} = 2.337[-((-0.026924)(-0.078507)) = -0.004940 \]

\[ B_{22} = 2.337[+(0.078507)^2] = 0.014406 \]

Check \( P_{TL} \)

\[ P_{TL} = B_{11} P_{G_1} + 2B_{12} P_{G_2} P_{G_1} + B_{22} P_{G_2} \]

\[ = (0.001694)(1.3013)^2 - 2(0.004940)(0.078507)(1.82) + (0.014406)(1.82)^2 \]

\[ = 0.0310 \text{ (compared with 0.0313; the small error is due to the fact that the small angle changes were in the order of the load flow convergence criteria).} \]
(c) Solve for the economically optimum division of load considering losses. The penalty factors are

\[ PF_1 = \frac{1}{1 - (\frac{\partial P_T}{\partial P_G})} = \frac{1}{1 - 0.003388 P_{G_1} + 0.009881 P_{G_2}} \]

\[ PF_2 = \frac{1}{1 + 0.009881 P_{G_1} - 0.028811 P_{G_2}} \]

\[ \lambda_1 = \frac{200(P_{G_1} + 1)}{2 \lambda_1 + 2 \lambda_2} \]

\[ \lambda_2 = \frac{80 P_{G_2} + 260}{1 + 0.009881 P_{G_1} - 0.028811 P_{G_2}} \]

The problem is solved by trial and error using a programmable calculator. The results of three trials are

<table>
<thead>
<tr>
<th>( P_{G_1} )</th>
<th>( P_{G_2} )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( P_L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0313</td>
<td>1.8200</td>
<td>400.4</td>
<td>423.5</td>
<td>2.820</td>
</tr>
<tr>
<td>1.1100</td>
<td>1.7400</td>
<td>416.4</td>
<td>415.5</td>
<td>2.823</td>
</tr>
<tr>
<td>1.1060</td>
<td>1.7410</td>
<td>415.6</td>
<td>415.6</td>
<td>2.820</td>
</tr>
</tbody>
</table>

The general problem is too complex to solve by hand. A flow chart for computer implementation is shown in Figure 8.5. The problem may also be formulated and solved by Newton–Raphson methods.

The foregoing discussion applies basically to an all-thermal system; that is, one where all generating units can be assigned an operation cost function (C_i). If hydro units are available, there is no fuel cost associated with the unit output and hence a negligible C_i function. However, this does not mean that there are no restrictions on hydro output whatsoever. The typical restriction involves the volume of water that may be removed from a reservoir in a specific time period. Consider

\[ P_h = \eta H \rho \frac{dv}{dt} \]

(8.32a)

where

\( P_h \) = hydraulic turbine power in W.
\( \eta \) = hydraulic efficiency of turbine and penstock.
\( g \) = acceleration of gravity = 9.8 m/s^2.
\( H \) = difference in elevation of reservoir surface and turbine (head) in m.
\( \rho \) = mass density of water = 1000 kg/m^3
\( \frac{dv}{dt} \) = volume of water flow through turbine in m^3/s.

For a constant-head, constant-efficiency situation,

\[ P_h = K \frac{dv}{dt} \]

(8.32b)

\[ K = \eta H \rho \]

(8.32c)

so that

\[ \int \frac{dv}{K} = \frac{1}{K} \int P_h \, dt \]

(8.33a)
8.3 System Voltage Control

Viewing the power system as an electric circuit, the basic source of voltage must be from an active element. The only such elements are the generators. Thus, the basic voltage-controlling component is the generator. To put it another way, to control the system voltage, we must first consider controlling each generator voltage.

8.3.1 Generator Voltage Control

Recall from Chapter 6 that generator voltage control is achieved by controlling its dc field current. Consider a 1000-MW generator. If 0.1% of this power is required by the field circuit, this computes to 1 MW, which, for a field voltage of 500 V, translates into a field current of 2000 A.

We see, then, that the field circuit requires a high-power controllable dc source. Such a dc source, dedicated to producing the generator dc field current, is called an excitation system. One possible source of power for excitation is the generator itself. This arrangement is called self-excitation, and an example in simplified form is shown in Figure 8.6.

The dispatch of hydro units can be controlled by constraints other than fuel costs, the primary consideration being the total energy available in a specific time period. Considering this, and transmission losses from hydro sites, it is possible to derive an equivalent operating-cost function and include this in the economic dispatch problem in a manner similar to our treatment of thermal units.

In a real power system, the load is continually changing with time, tracking through a 24-hour load cycle. Speed control loops of the generators continuously operate to maintain constant system frequency. As these control adjustments are made, generator outputs will tend to drift away from their optimum economic dispatch settings. Every few minutes, the drift is sufficient to warrant the calculation and implementation of a fresh solution to the economic dispatch problem. The total system load at a given point in time can be determined from measured values for all generator outputs and tie line flows. These data are collected and transmitted to a central location (usually called an energy control center), where the economic dispatch problem is solved in real time on a dedicated energy control computer. The optimum generation allocations are transmitted back to the respective generator sites, where the prime-mover outputs are adjusted, typically in five-minute intervals. Physical control of the prime-mover turbines is achieved by adjusting the main steam valves, in the case of thermal units, or the penstock gates, in the case of hydro units.

\[ V = \frac{1}{k} W_q \]  

(8.33b)

Thus, the volume \( V \) of water we draw from the reservoir is directly proportional to the energy \( W_q \) we deliver to the system.

Example 8.4

A potential site for hydroelectric generation has a 100-m head. Assuming a hydraulic efficiency of 100%, how much water flow in m³/s must flow through the turbine to generate 40 MW?

Solution

\[ P_u = 40 \times 10^6 = (1)(9.8)(1000)(100) \frac{du}{dt} \]

\[ \frac{du}{dt} = 40.8 \text{ m}^3/\text{s} \]