Questions from last lecture?
How are your matrix math skills?
Questions on Hmwk #1?
Everybody all set w/text, matlab, etc?
Web page ok? Be sure to get 3-ring binder - lots of papers!

TODAY:
- Kron Reduction (network reduction) – refer to Lecture 2.
- Modifications to [Y] according to system changes
- Switching things in and out: lines, shunt reactors/caps, gens.
- Augmenting the admittance matrix
  - Ungrounded Voltage Source
  - Short Circuit
  - Two-winding Transformer
- Next: in situ methods to solve $[Y_{bus}] [V_{bus}] = [I_{inj}]$
Admittance Equations

General Form:

\[
\begin{bmatrix}
Y_{\text{Bus}}
\end{bmatrix}
\begin{bmatrix}
V_{\text{NODE}}
\end{bmatrix}
= 
\begin{bmatrix}
I_{\text{INJ}}
\end{bmatrix}
\]

We can add constraints:
- \text{V Source Bus-Bus}
- \text{Short}
- \text{XFMR}
- \text{Dependent Sources (OP-AMP)}
SWITCHING LINES, ETC., in and out:

From Lecture 1:

\[ [Y] = \begin{bmatrix} 8 & -2 & -5 \\ -2 & 9 & -4 \\ -5 & -4 & 15 \end{bmatrix} \]

Switch out Line 1-3:

\[ \begin{bmatrix} 3 & -2 & 0 \\ -2 & 9 & -4 \\ 0 & -4 & 10 \end{bmatrix} \]

\[ y_{1-3} = 55 \]
\[
\begin{align*}
\bar{y}_n &= \bar{y}_n - \bar{y}_{n-3} \\
\bar{y}_{13} &= \bar{y}_{13} + \bar{y}_{1-3} \\
\bar{y}_{31} &= \bar{y}_{31} + \frac{\bar{y}}{\bar{y}_{3-1}} \\
\end{align*}
\]

How about a T-Line?

\[
\begin{align*}
K & \quad R \quad jX_L \quad n \\
\text{Ref} \\
\end{align*}
\]

Modify [Y], switch out...?

Be a full-line charging susc.

\[
\begin{align*}
\bar{y}_{KK} &= \bar{y}_{KK} - \bar{y}_K - \bar{y}_{K-N} \\
\bar{y}_{Kn} &= \bar{y}_{Kn} + \bar{y}_{K-N} \\
\bar{y}_{Kn} &= \bar{y}_{Kn} + \bar{y}_{n-K} \\
\bar{y}_{nK} &= \bar{y}_{nK} + \bar{y}_{n-K} \\
\end{align*}
\]
AUGMENTING $[Y_{bus}]$

$Y_{bus}$

$100000$

$0.0000$

$V_{bus}$

$X$

$V_{bus} = I_{bus}$

Coefficients of add'l eqns

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Node-Node Voltage Source

\[ V_s = V_e - V_k \]

New Constraints being added:

1. \[ V_s = V_e - V_k = \text{Known (forced voltage)} \]
2. \[ I_e = + I_s \]
3. \[ I_k = - I_s \]
\[ \tilde{V}_s = \tilde{V}_k - \tilde{V}_k \]

Note:

- This effect is on the left side of eqn!
- Change Sign when entering in Column N+1 of \([N]\)

\( \tilde{I}_k(\text{INJ}) \) is increased by \( +\tilde{I}_s \)

\( \tilde{I}_k(\text{INJ}) \) is "increased" by \( -\tilde{I}_s \)

IF \( \tilde{V}_s = 0 \) ⇒ SHORT CIRCUIT

K-2

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The turns ratio \( a \) is in general, complex.

Polarity Marks:
\[
\begin{align*}
\frac{L_{p}}{L_{s}} & = \frac{V_{p}}{V_{s}} \\
L_{mk} & = \sqrt{L_{p}L_{s}}
\end{align*}
\]
Aug Row \rightarrow Voltage constraints
Aug Col \rightarrow Current constraints

TYPICAL: INTRODUCE ZERO IN MAIN DIAG.
**Constraints**

**Node Voltages**

\[ \tilde{V}_{m} - \tilde{V}_{k} = a(\tilde{V}_{e} - \tilde{V}_{j}) \]

i.e. \[ \tilde{V}_{mk} = a\tilde{V}_{ej} \]

\[ \tilde{V}_{m} - \tilde{V}_{k} + a\tilde{V}_{j} - a\tilde{V}_{e} = 0 \]

**Injected Currents:**

\[
\begin{align*}
  l: & +i_{s} = aI_{p} \\
  j: & -i_{s} = -aI_{p} \\
  m: & -I_{p} \\
  k: & +I_{p}
\end{align*}
\]

Again: effect is on left side of equation. Change signs in Column No. 1 of [47].

General Reference for this is on next page. (From Vlach - refer to syllabus reference list.)
<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>SYMBOL</th>
<th>MATRIX</th>
<th>EQUATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CURRENT SOURCE</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$I = \begin{bmatrix} I_1 \ I_2 \end{bmatrix}$</td>
</tr>
<tr>
<td>VOLTAGE SOURCE</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$V = \begin{bmatrix} V_1 \ V_2 \end{bmatrix}$</td>
</tr>
<tr>
<td>OPEN CIRCUIT</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$V = V_1 - V_2$</td>
</tr>
<tr>
<td>SHORT CIRCUIT</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$V_1 - V_2 = 0$</td>
</tr>
<tr>
<td>ADMITTANCE</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$I = \begin{bmatrix} I_1 \ I_2 \end{bmatrix}$</td>
</tr>
<tr>
<td>IMPEDANCE</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$V_1 - V_2 = 0$</td>
</tr>
<tr>
<td>NULLATOR</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$V_1 = V_2$</td>
</tr>
<tr>
<td>NORATOR</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$V, I$ are arbitrary</td>
</tr>
<tr>
<td>VCT</td>
<td>![Symbol]</td>
<td>![Matrix]</td>
<td>$I = \begin{bmatrix} I_1 \ I_2 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

**Fig. 4.4.1.** Ideal elements in the modified nodal formulation without graphs.