Combinatorial Probability Review

- Given Two Events A & B.
  - *Independent* ($A \perp B$) ⇒
    * Status of A or B has no impact on status of the other
    * Knowledge of A give no information about B (and vice versa)
  - *Mutually Exclusive (Mutex)*) ⇒
    * If A is true, then B must be false (and vice versa)
    * No more than one of A & B can be true at any given time
  - *Collectively Exhaustive (Collex)*) ⇒
    * A & B cover the entire space of all possibilities
    * At least one of A & B must be true at all times
  - *Mutually Exclusive and Collectively Exhaustive (MECE)*) ⇒
    * A & B Partition the space of all possibilities
    * Exactly one of A & B must be True at all times

- Conditional Probabilities for (A & B)
  - $Pr(A \mid B) = Probability$ that A is True, given that B is True
  - *Probabilities in Specific Cases:*
    * Generic: $Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$
    * Indep: $Pr(A \mid B) = \frac{Pr(A) \times Pr(B)}{Pr(B)} = Pr(A)$
    * Mutex: $Pr(A \mid B) = 0$
    * Collex: $Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} = generic$
    * MECE: $Pr(A \mid B) = 0 = mutex$
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• Intersection of Two Events (A & B)

  • \( (A \cap B) = (A \land B) = \)

  \textit{Event in which both A and B are true}

  • Probabilities in Specific Cases:

    * Generic: \( \Pr (A \cap B) = \Pr(A) \cdot \Pr(B|A) \)
      \( = \Pr(B) \cdot \Pr(A|B) \)

    * Indep: \( \Pr (A \cap B) = \Pr(A) \cdot \Pr(B) \)

    * Mutex: \( \Pr (A \cap B) = 0 \)

    * Collex: \( \Pr (A \cap B) = \Pr(A) \cdot \Pr(B|A) = \text{generic} \)

    * MECE: \( \Pr (A \cap B) = 0 = \text{mutex} \)

• Union of Two Events (A & B)

  • \( (A \cup B) = (A \lor B) = \)

  \textit{Event in which A or B or both are true}

  • Probabilities in Specific Cases:

    * Generic: \( \Pr (A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) \)
      \( = \Pr(A) + \Pr(B) - \Pr(A)\Pr(B|A) \)

    * Indep: \( \Pr (A \cup B) = \Pr(A) + \Pr(B) - \Pr(A)\Pr(B) \)

    * Mutex: \( \Pr (A \cup B) = \Pr(A) + \Pr(B) \)

    * Collex: \( \Pr (A \cup B) = 1.0 \)

    * MECE: \( \Pr (A \cup B) = \Pr(A) + \Pr(B) = 1.0 \)