The target is a distance $d_0$ away from the radar system, and is moving with constant velocity $N$.

(If the target is moving toward the radar, then $N$ is positive; if away then $N$ is negative.)

The transmitted signal is $s(t)$.

The received signal is approximately

$$f(t) = s(t - \frac{2d(t)}{c})$$

Where $d(t)$ is the distance from the radar to the target:

$$d(t) = d_0 - Nt,$$

And $c$ is the propagation speed for the radar pulse, $c = 3 \times 10^8$ m/s.
Suppose \( s(t) = \text{RECT}(t/T) \cos(2\pi f_0 t) \).

Then
\[
\Gamma(t) = \text{RECT}(t - \frac{2\lambda_0}{c} + \frac{2\lambda_0 f_0}{c}) \cos\left(2\pi f_0 t - 2\pi \frac{2\lambda_0}{c} + 2\pi \frac{2\lambda_0 f_0}{c} t\right)
\]
\[
\approx \text{RECT}(t - \frac{2\lambda_0}{c}) \cos\left(2\pi f_0 (1 + \frac{2\lambda_0}{c}) t - 2\pi f_0 \frac{2\lambda_0}{c}\right)
\]

**Comments**

1. The pulse is delayed by \( \frac{2\lambda_0}{c} \) seconds.

2. The frequency shift for the \( \cos \) is
\[
f_{\text{shift}} = f_0 \left(1 + \frac{2\lambda_0}{c}\right) = f_0 + \frac{2\lambda_0 f_0}{c}
\]

3. The phase shift for the \( \cos \) is
\[
\phi = -2\pi f_0 \frac{2\lambda_0}{c}
\]
(This is LARGELY UNIMPORTANT.)

**Example**

Suppose \( \lambda_0 = 1 \) km and \( v = 125 \) km/hr. If the radar frequency is \( f_0 = 10 \) GHz.

The pulse delay will be
\[
\frac{2\lambda_0}{c} = \frac{2 \times 10^3}{3 \times 10^8} = 6.67 \times 10^{-6} \text{ s}
\]
\[
= 6.67 \mu\text{s},
\]

and the frequency shift will be
\[
\frac{2\lambda_0 f_0}{c} = \frac{2 \times 125 \times 10^9}{3 \times 10^8} \left(\frac{1 \text{ km}}{3600 \text{ sec}}\right) \times 10 \times 10^9
\]
\[
= 2.35 \text{ kHz}
\]
BASEBAND Processing:

\[ \Gamma(t) \xrightarrow{\times} LPF \rightarrow \Gamma_I(t) \]

\[ \cos(2\pi f_0 t) \]

\[ -\sin(2\pi f_0 t) \]

\[ \times \]

\[ \Gamma(t) \xrightarrow{\times} LPF \rightarrow \Gamma_Q(t) \]

LPF: "Low Pass Filter"

In general, suppose \( \Gamma(t) = P(t - \gamma) \cos(2\pi [f_0 + \Delta f] t + \phi) \)

\[ \Gamma(t) \cos(2\pi f_0 t) = \frac{P(t - \gamma)}{2} \cos(2\pi [f_0 + \Delta f] t + \phi) \cos(2\pi f_0 t) \]

\[ = \frac{P(t - \gamma)}{2} \cos(2\pi \Delta f t + \phi) \]

\[ + \frac{P(t - \gamma)}{2} \cos(2\pi [2f_0 + \Delta f] t + \phi) \]

\[ \implies \Gamma_I(t) = \frac{P(t - \gamma)}{2} \cos(2\pi \Delta f t + \phi) \]

\[ \Gamma(t) \sin(2\pi f_0 t) = \frac{P(t - \gamma)}{2} \sin(2\pi [f_0 + \Delta f] t + \phi) \sin(2\pi f_0 t) \]

\[ = \frac{P(t - \gamma)}{2} \sin(2\pi \Delta f t + \phi) \]

\[ - \frac{P(t - \gamma)}{2} \sin(2\pi [2f_0 + \Delta f] t + \phi) \]

\[ \implies \Gamma_Q(t) = \frac{P(t - \gamma)}{2} \sin(2\pi \Delta f t + \phi) \]

Two channels are needed to resolve the sign of \( \Delta f \).