EE2150 Exam 2, Fall 2010

There are 4 equally-weighted questions on this exam. You must show your work for full credit.

1. Given \( x[n] = 7e^{j(0.1\pi n + 0.25\pi)} \) and the filter response: \( H(z) = 1 + z^{-5} \), if the output is written as: \( Ae^{j(\hat{\omega}n + \phi)} \), find the numerical values of \( A, \hat{\omega}, \) and \( \phi \).

   Solution:
   \[
   H(0.1\pi) = 1 + e^{-j0.5\pi} = (1 - j) = \sqrt{2}e^{-j\pi/4}
   
   Therefore, \( y[n] = \sqrt{2}7e^{j0.1\pi n + 0.25\pi - \pi/4} \) or:
   
   \( A = 7\sqrt{2} \)

   \( \hat{\omega} = 0.1\pi \)

   \( \phi = \pi/4 - \pi/4 = 0 \)

2. If \( H(\hat{\omega}) = e^{-j\hat{\omega} \cos(\hat{\omega})} \) and \( x[n] = 8\cos(n\pi/3 - \pi/6) \), find the output \( y[n] \).

   Solution:
   \[
   H(\pi/3) = e^{-j\pi/3 \cos(\pi/3)} = 0.5e^{-j\pi/3}
   
   Therefore:
   
   \( y[n] = 4\cos(n\pi/3 - \pi/2) \)
3. Given: \( y[n] = (x[n] + x[n-1] + x[n-2])/3 \), find the poles and zeroes in the z-domain.

Solution:
\[ \mathcal{H}(z) = (1/3)(1 + z^{-1} + z^{-2}) = (1/3)\frac{(z^2 + z + 1)}{z^2} \]
Therefore, 2 poles @ \( z=0 \) and zeroes @ \( z = e^{\pm j(2\pi/3)} \)

4. For an LTI system described by \( y[n] = x[n] + x[n-4] \), sketch and label a plot of \(|\mathcal{H}(\omega)|\) for \(-\pi < \omega < \pi\).

Solution:
\[ |\mathcal{H}(\omega)| = |1 + e^{j4\omega}| = |e^{-j2\omega}2\cos(2\omega)| = |2\cos(2\omega)| \]