EE3140 Hour Exam 1, Fall 2011

Note that the problems have different point values. All units are mks. Show your work for full credit. Useful constants:

\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}, \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \]

1. Given an interface with unit normal \( \hat{z} \) and \( \vec{D}_2 = \hat{z} - \hat{z} \) [Coul/m²] for \( z < 0 \) and \( \vec{D}_1 = +\hat{z} \) [Coul/m²] for \( z > 0 \), circle the answer below that best describes this situation: (3 points)

(a) medium 1 and medium 2 are dielectrics with \( \varepsilon_1 > \varepsilon_2 \)
(b) medium 1 and medium 2 are dielectrics with \( \varepsilon_1 < \varepsilon_2 \)
(c) there is positive surface charge on the boundary between two dielectrics
(d) medium 2 is a perfect conductor
(e) Impossible

2. Given an electric field: \( \vec{E} = 5 \exp^{j\beta y} \hat{x} \) and the medium is free space, find the corresponding magnetic field. Find the frequency, \( f \), and direction of propagation.

\[ \vec{H} = \frac{\nabla \times \vec{E}}{j \omega \mu_0} = \hat{x} \frac{4 \pi \varepsilon_0}{\omega \mu_0} \exp^{j \beta y} = 0.0133 \hat{x} \exp^{j \beta y} \]

\[ \vec{H} = \frac{0.133}{2} \exp^{j \beta y} \left[ \frac{\beta}{\mu_0} \right] \] (4 points)

\[ f = \frac{382}{\mu_0} \text{ Hz} \] (4 points)

direction of propagation = \( -y \) direction (2 points)

\[ k = \beta = \frac{\omega}{c} = \frac{2\pi f}{c} \quad \Rightarrow \quad f = \frac{c k}{2\pi} = \frac{3 \times 2 \times 10^8}{2\pi} \]

From \( \vec{E} = 5 \exp^{j\beta y} \hat{x} \), \( E(x) = x 5 \cos (8y + \omega t) \)

\[ -y \text{- direction} \]
3. Given a magnetic field \( \mathbf{H} = 150 \exp \left( \frac{j \pi}{2} \right) \), find the power (in Watts) passing through a square in the z=0 plane, 2 meter on each side (5 points).

\[
\mathbf{\mathbf{S}} = \frac{1}{2} \left\{ \mathbf{E} \times \mathbf{H}^* \right\} = \frac{1}{2} \left[ \mu_0 / |\mathbf{H}|^2 \right]
\]

\[
= \frac{1}{2} \left( 1.25 \right)^2 \times \frac{377}{2} \left[ \text{W/m}^2 \right]
\]

\[
= 4.24 \times 10^{-2} \left[ \text{Watts/m}^2 \right]
\]

Through 4 m² gives:

Power = 1696 Watts

4. Given that an electric field strength of 1.0 V/m at a frequency of 1 kHz exists just below the surface of seawater (\( \sigma = 4 \text{ mhos/m} \) and \( \varepsilon_r = 81 \)), find the signal strength 10 meters below the surface. (5 points)

\[
|\mathbf{E}| = E_0 e^{-kz}
\]

\[
k_z = \left| \text{Im} \left\{ \omega \mu_0 / (81 i \varepsilon_0) \right\} \right|
\]

\[
= \left| \text{Im} \left\{ 2 \pi 1000 \left( -\frac{4 \pi \times 10^{-7} \times 81}{8.85 \times 10^{-12} - \frac{9}{2 \pi 1000} } \right) \right\} \right|
\]

\[
= \left| \text{Im} \left\{ 0.125664 - j 0.125664 \right\} \right|
\]

\[
= 0.125664
\]

\[
|\mathbf{E}|_{z=10} = 1.0 e^{-0.126(10)}
\]

\[
= 0.28 \left[ \text{V/m} \right]
\]
5. A uniform plane wave, \( \vec{E} = \hat{y}E_0 e^{-j k z} \), with normal incidence to a dielectric interface with surface normal \( \hat{n} = \hat{z} \) has a total electric field, \( |E_{y, \text{total}}| \), as shown in the figure below.

(a) Given that the left-hand side (medium 1) is free space, find the permittivity of the medium 2. (4 points)

\[
R_1 = \frac{k_2 - k_1 e^{-j \kappa z}}{k_2 e^{-j \kappa z} - k_1} = \frac{1 - \sqrt{\varepsilon_{r_2}}}{1 + \sqrt{\varepsilon_{r_2}}}
\]

\[
E_1 \text{-field in region } 1 \quad \tau_5 = \frac{E_0}{1 + R_1 e^{2j \kappa z}}
\]

At \( z = 0 \), \( |1 + R_1| = \frac{2}{3} \quad \Rightarrow \quad R_1 = -\frac{1}{3}
\]

So \( \frac{1}{3} = \frac{1 - \sqrt{\varepsilon_{r_2}}}{1 + \sqrt{\varepsilon_{r_2}}} \quad \Rightarrow \quad \varepsilon_{r_2} = 4
\)

(b) Find the wavelength of the wave in medium 2. (4 points)

\[
\frac{\lambda}{2} = (4 - 5 - 105) = 3m
\]

\[
\Rightarrow \lambda = 6m
\]