Note that the problems have different point values. All units are mks. Show your work for full credit. Useful constants:

\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \]

1. Given an electric field: \( \vec{E}(y, t) = 20.3 \cos(2t + 4y) \), find the magnetic field, \( \vec{B} \), the frequency, \( f \), and the direction of propagation.

\[ \vec{E} = \hat{y} 0.3 \cos 2y \Rightarrow \nabla \times \vec{E} = -j \omega \varepsilon_0 \vec{E} = -2(0.3) j 2 \cos 4y \Rightarrow \vec{B} = 0.3 \frac{k}{\omega} \hat{y} \hat{z} \]
\[ \vec{B}(y, t) = \Re \left( \vec{E} e^{-j \omega t} \right) = 0.3 \frac{k}{\omega} \cos (2t + 4y) \]
\[ \vec{B} = 0.16 \cos (2 \pi + 4y) \]
\[ \omega = 2\pi \left[ \sqrt{1 + \frac{B_0}{\mu_0}} \right] \text{ Tesla (or} \frac{A}{m^2} \text{)} \]
\[ f = \frac{\omega}{2\pi} \]
\[ \text{Direction of propagation} = -\hat{y} \]

2. Find the polarization (linear, circular, or elliptical) for the following electric fields (3 points for each):

(a) \( \vec{E} = [(1 + j)\hat{y} + (1 - j)\hat{z}] \exp(-jkr) \)

magnitude of \( \hat{y} \) \& \( \hat{z} \) the same, 90° out of phase

\[ \Rightarrow \text{Circular} \]

(b) \( \vec{E} = [(2 + j)\hat{x} + (3 - j)\hat{z}] \exp(-jky) \)

magnitude and phase of \( \hat{x} \) \& \( \hat{z} \) not equal

\[ \Rightarrow \text{Elliptical} \]
3. A wave with polarization parallel to the plane of incidence at an angle of 30° relative to the normal has \( k_1 = 19.18 \). If region 2 has \( \varepsilon_2 = (51 - j31.43)\varepsilon_0 \), find \( k_2 \), the propagation constant in the z-direction in region 2. (5 points)

\[
K_1 = k_1 \sin 30° = (0.5)(19.18)
\]

\[
k_2 = \frac{\omega \sqrt{\mu_0 \varepsilon_2}}{c}
\]

\[
= 2\pi (9.15 \times 10^9) \sqrt{\frac{\pi \times 10^{-9}}{51 - j31.43}}
\]

\[
= 142.8 - j40.47
\]

\[
k_{z2} = \sqrt{k_2^2 - k_x^2}
\]

\[
= \frac{142.8 - j40.47}{\sqrt{11}}
\]

4. Ice has a conductivity of roughly \( \sigma = 10^{-6} \text{mho/m} \) and \( \varepsilon = 3.2\varepsilon_0 \). If an electromagnetic wave impinges at normal incidence to a block of ice, how far from the surface must one go for the field to reach \( 1/e \) (or 37%) of the value at the surface? (5 points)

\[
K = \frac{\omega}{\sqrt{\mu_0 \varepsilon}} (1 - j \frac{\sigma}{\omega \varepsilon})
\]

\[
= \frac{2\pi (10^9)}{c} \sqrt{\frac{3.2}{1 - j} \frac{10^{-6}}{(2\pi \times 10^9) (3.2 \varepsilon_0)}}
\]

\[
= 37.49 - j0.0001093
\]

\[
\Rightarrow \frac{(\frac{1}{\text{Im}[K]})^2}{\text{Im}[K]} = e^{-1} \Rightarrow Z = \frac{1}{\text{Im}[K]} = 9.5 \text{ km}
\]

Note: For \( \sigma / \omega \varepsilon \ll 1 \),

\[
K = \omega \sqrt{\mu_0 \varepsilon} (1 - j \frac{\sigma}{\omega \varepsilon})
\]

\[
d_p = \frac{1}{\text{Im}[K]} = \frac{\varepsilon}{\mu} \quad \text{(independent of \( \omega \))}
\]