Two-digit number \( \mathcal{K} \in \mathcal{Y} \)

EE3140 Hour Exam 1, Fall 2014

Note that the problems have different point values. All units are mks. Show your work for full credit.

1. Given an electric field: \( \vec{E}(y, t) = 0.5 \cos(4t - 4y) \), find the magnetic field, \( \vec{B}(t) \), the frequency, \( f \), and the direction of propagation.

\[
\nabla \times \vec{E}(y, t) = -\frac{\partial \vec{B}}{\partial t} = 2 \frac{\partial}{\partial y} \sin(4t - 4y) \quad \text{(Take)} \quad \left| \begin{array}{ccc}
\hat{x} & \hat{y} & \hat{z} \\
0 & 2 & 0 \\
-\frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z}
\end{array} \right|
\]

\[
\Rightarrow \quad \frac{\partial}{\partial y} = \int \left( -\frac{\partial}{\partial y} \right) dt
\]

\[
= -\frac{1}{4} \cos(4t - 4y) \left[ T_{z, \phi} \right]
\]

\[
\vec{B} = -\frac{1}{2} \cos(4t - 4y) \left[ T_{z, \phi} \right] \quad (5 \text{ points})
\]

\[
f = \frac{2}{\pi} = \frac{1}{\sqrt{1 + \left( \frac{2}{\phi} \right)^2}} \quad (6 \text{ points})
\]

direction of propagation = \( +\frac{\hat{y}}{2} \) \( (4 \text{ points}) \)

2. Given an interface with unit normal \( \hat{z} \) and \( \vec{D}_2 = 2\hat{x} + \hat{z} \ [\text{Coul/m}^2] \) for \( z < 0 \) and \( \vec{D}_1 = \hat{x} + \hat{z} \ [\text{Coul/m}^2] \) for \( z > 0 \), circle the answer below that best describes this situation: (8 points)

(a) medium 1 and medium 2 are dielectrics with \( \varepsilon_1 > \varepsilon_2 \)

(b) medium 1 and medium 2 are dielectrics with \( \varepsilon_1 < \varepsilon_2 \)

(c) there is positive surface charge on the boundary between two dielectrics

(d) medium 2 is a perfect conductor

(e) impossible

\[
\vec{D}_2 = 2; \quad \vec{D}_1 = 1
\]

\[
= \varepsilon_2 \vec{E}_2 \quad = \varepsilon_1 \vec{E}_1
\]

From \( \vec{E}_1 = \vec{E}_2 \)

\[
\Rightarrow \quad \frac{1}{\varepsilon_1} = \frac{2}{\varepsilon_2} \quad \Rightarrow \quad \varepsilon_L = 2 \varepsilon_1
3. A wave with frequency 100 MHz and polarization parallel to the plane of incidence impinges on an interface at an angle of 30° relative to the normal. Given \( k_1 = 6k_0 \) in region 1 where \( k_0 = \omega \sqrt{\mu_0 \epsilon_0} \), if region 2 has \( \epsilon_2 = 25\epsilon_0 \), find \( k_{z2} \), the propagation constant in the z-direction in region 2 in terms of \( k_0 \). (8 points)

\[
\begin{align*}
K_{x1} &= 6k_0 \\
\beta_1 &= \sqrt{\epsilon_1} = 1 \\
\beta_2 &= \sqrt{\epsilon_2} = \sqrt{25} \\
K_{x2} &= \frac{5k_0}{3k_0} = 4k_0 \\
K_{z2} &= \sqrt{25k_0^2 - 9k_0^2} = 4k_0 \\
K_{x1} &= 3k_0
\end{align*}
\]

4. Ice has a conductivity of roughly \( \sigma = 10^{-6} \text{mho/m} \) and \( \epsilon = 3.2\epsilon_0 \). If an electromagnetic wave with frequency \( f = 1 \text{MHz} \) impinges at normal incidence to a block of ice, how far from the surface must one go for the field to reach \( 1/e \) (or 37%) of the value at the surface? (8 points)

\[
k = \omega \sqrt{\mu_0 \epsilon (1 - j \frac{\sigma}{\omega \epsilon})}
\]

\[
= 0.037 - j 0.001053
\]

\[
\text{or } e^{-1} = \frac{I_m[r]}{I_m[r]} = e^{-1}
\]

\[
\Rightarrow \frac{1}{I_m[r]} = 9.5 \text{ km}
\]