EE3140 Hour Exam 1, Spring 2002

Note that the problems have different point values. All units are mks. Show your work for full credit. Useful constants:

\[
\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \\
\mu_0 = 4\pi \times 10^{-7} \text{ H/m}
\]

1. Given an interface with unit normal \( \hat{z} \) and \( \vec{D}_2 = -\hat{z} \, [\text{Coul/m}^2] \) for \( z < 0 \) and \( \vec{D}_1 = +\hat{z} \, [\text{Coul/m}^2] \) for \( z > 0 \), circle the answer below that best describes this situation: (3 points)

(a) medium 1 and medium 2 are dielectrics with \( \varepsilon_1 > \varepsilon_2 \)
(b) medium 1 and medium 2 are dielectrics with \( \varepsilon_1 < \varepsilon_2 \)
(c) there is positive surface charge on the boundary between two dielectrics
(d) medium 2 is a perfect conductor
(e) impossible

2. Given a magnetic field: \( \vec{H} = 5 \exp^{j8\pi y} \hat{x} \) and the medium is free space, find the corresponding electric field. Find the frequency, \( f \), and direction of propagation.

\[
\vec{E} = \frac{\nabla \times \vec{H}}{j \omega \varepsilon_0} = -5 \frac{j8\pi y}{\omega} \hat{z} \\
f = \frac{c}{\lambda} = \frac{c}{2\pi} = \frac{382 \, \text{MHz}}{2} \\
\text{direction of propagation} = -\hat{j}
\]
3. Given an electric field \( \mathbf{E} = 150 \exp^{j10\pi} \), find the power (in Watts) passing through a square in the \( z=0 \) plane, 1 meter on each side (5 points).

\[
\langle \mathbf{S} \rangle = \frac{1}{2} \mathbf{E} \times \mathbf{H}^* = \frac{1}{2} \frac{|\mathbf{E}|^2}{\eta_0} = \frac{1}{2} \frac{(150)^2}{377} = 29.8 \, \text{W} \frac{\text{Hz}}{\text{m}^2}
\]

\( S_0, \quad 1 \, \text{m}^2 \Rightarrow \text{Power} = 29.8 \, \text{W} \frac{\text{Hz}}{\text{m}^2} \)

4. Given that an electric field strength of 7.43V/m at a frequency of 1MHz exists just below the surface of seawater (\( \sigma = 4 \) mhos/m and \( \epsilon_r = 81 \)), find the distance, \( d \), that a submarine can be submerged and still detect a signal if the submarine's receiver has a minimum detectable field of 10\( \mu \)V/m (5 points).

**Given:**

\[
E_{\text{H}_2\text{O}} = E_0 e^{-k_z z}
\]

\[
\frac{10 \mu \text{V}}{\text{m}} = 7.43 \frac{\text{V}}{\text{m}} e^{-k_z d}
\]

\[
k_z = \left| \text{Im} \left[ \omega \mu_0 (81 \epsilon_0 - \frac{\sigma}{\omega}) \right] \right| = 3.9716
\]

\( S_0, \quad d = \left( - \ln \frac{10 \times 10^{-6}}{7.43} \right) \frac{1}{3.9716} = 3.41 \, \text{m} \)
5. Find the polarization (linear, circular, or elliptical and left- or right-hand) for the following electric fields (2 points for the type, 1 point for the handed-ness):

(a) \( \vec{E} = [(1 + j)\hat{y} + (1 - j)\hat{z}] \exp(-j k x) \)

\[ \text{Circulally polarized, right-handed} \]

(b) \( \vec{E} = [(2 + j)\hat{x} + (3 - j)\hat{z}] \exp(-j k y) \)

\[ \text{Elliptically polarized, left-handed} \]

6. A uniform plane wave, traveling in the z-direction, with normal incidence to a dielectric interface has a total electric field, \( |E_{\text{y, total}}| \), as shown in the figure below.

(a) Given that the left-hand side (medium 1) has \( \varepsilon_1 = 9\varepsilon_0 \), find the permittivity of the medium 2. (4 points)

\[
\begin{align*}
M_{nx} : & \quad |1 + |R_x^2| = 1.5 \\
M_{nz} : & \quad |1 - |R_z^2| = 0.5
\end{align*}
\]

A + Z = 0 , \( |E_{\text{y, total}}| = 1.5 = \sqrt{1 + \frac{3\varepsilon_0 - \varepsilon_2}{3\varepsilon_0 + \varepsilon_2}} \)

\[ \Rightarrow \varepsilon_2 = \varepsilon_0 \]

(b) Find the wavelength of the wave in medium 2. (4 points)

\[
\begin{align*}
\lambda_1 &= 1 \text{ m} \\
k_1 &= \frac{2\pi}{\lambda_1} = \frac{2\pi}{1} = \omega \sqrt{\mu_0 \varepsilon_1} \\
k_2^2 &= \frac{2\pi}{\lambda_2} = \omega \sqrt{\mu_0 \varepsilon_2} \\
\Rightarrow \lambda_2 &= \frac{\lambda_1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \text{ m} \\
\Rightarrow \lambda_2 &= \frac{1}{3.826} \text{ m} = \frac{c}{100 \text{ MHz}} = 3 \text{ m} \text{ (as a check)}
\end{align*}
\]