EE3140 Final Exam, Fall 2014

There are 5 problems, with points per problem as stated. All units are mks. Show your work for full credit.

1. A transmitting short dipole is positioned at the origin, along the z (vertical) axis. A second short dipole has its center on the y-axis 2km distance from the first dipole, is also aligned with the z-axis, and an voltage of 1mV is measured. The second dipole is then elevated 1km (placing its center at x=0, y=2km, z=1km), and aligned with the z-axis. What is the new voltage measured at the second dipole? (5 points)

\[
\frac{\text{Field/voltage drops by a factor of}}{V_i} = \frac{\sin \theta}{(r_i/r_o)} \sin \theta
\]

\[
\theta = 90^\circ = \pi/2
\]

\[
= 6.34 \times 10^{-1} \quad \text{V}_i = 0.894
\]

\[
\frac{V_o}{V_i} = 0.357 \text{ mV}
\]

\[V = 0.357 \text{ mV}\]

2. A uniform plane wave in air, traveling in the z-direction, with normal incidence to a dielectric wall has a total electric field, \(E^{\text{total}}\), as shown in the figure below. (6 points)

\[E_{\text{air}} = |E_{\text{air}}| = |E + n|\]

\[E_{\text{wall}} = 0 \quad z = 0 \quad \frac{1}{L} = |1 + R| \]

\[R = \frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon_L}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon_L}} = \frac{1}{2} \left( \frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon_L}}{\varepsilon_0 - \sqrt{\varepsilon_L}} \right)\]

\[\frac{1}{2} \sqrt{\varepsilon_L} = \frac{3}{2} \sqrt{\varepsilon_0}\]

\[\Rightarrow \varepsilon_L = \frac{9}{2} \varepsilon_0\]

What is the permittivity, \(\varepsilon\), of the dielectric wall?
3. Two isotropic oscillators are placed on the y-axis, at \( y = -\lambda/4 \) and \( y = \lambda/4 \), with an intrinsic phase shift of \( \psi = \pi \). Given that the azimuthal angle, \( \phi \), is defined as the angle from the x-axis, determine all values of \( \phi \) which give nulls for the electric field pattern in the x-y plane. (6 points)

\[
\begin{align*}
\theta &= \frac{\lambda}{2} ; \\
\phi &= \frac{\pi}{2} ; \\
\cos \gamma &= \sin \phi \\
\sin \left( \frac{\pi \sin \phi + \pi}{2} \right) \\
\end{align*}
\]

Find zeros at:

\[
\frac{\sin \left( \frac{\pi \sin \phi + \pi}{2} \right)}{2 \sin \left( \frac{\pi \sin \phi + \pi}{2} - \phi \right)}
\]

Out of phase along \( \pm x \) axis \( \rightarrow \)

\( \phi = 0, \pi \)

nulls = \( 0, \pi \)

4. A dipole of length \( 2\lambda \) is oriented along the \( z \)-axis. Determine all values of \( \theta \) which give nulls in the electric field pattern. (6 points)

\[
\begin{align*}
|\mathbf{E}| &= \cos \left( \frac{k \theta}{2} \cos \phi \right) \cos \frac{k \lambda}{2} = \frac{\cos (2\pi \cos \theta) - 1}{\sin \theta} \\
\text{for } k\lambda &= \pi/2 \\
\text{Try } \theta &= 0 \text{, } |\mathbf{E}| = 0 \Rightarrow \text{L'Hopital's rule:} \\
\frac{2}{\sin \theta} \left( \cos (2\pi \cos \theta) \right) \\
\frac{\cos \theta}{\sin \theta} \\
\Rightarrow 0 \text{ as } \theta \rightarrow 0 \text{ or } \pi \\
\end{align*}
\]

nulls = \( 0, \pm \frac{\pi}{2}, \pi \) (four nulls)
5. Two half-wavelength dipoles are aligned parallel to the y-axis, but have their centers located at $x = \pm \lambda/4$ on the x-axis (so that they are separated by $\lambda/2$). Assuming that they are driven in phase, with equal amplitudes. For the x-y plane, on the axes provided, sketch the element pattern, the array factor and the total (far-field) pattern (using pattern multiplication) (3 points for each sketch). Note that the sketches must show the correct placement of the nulls, but need not otherwise be quantitatively perfect.