EE3140 Final Exam, Spring 2017

There are 5 problems, with points per problem as stated. All units are mks. Show your work for full credit.

1. A transmitting short dipole (\( \lambda >> l \)) is positioned at the origin, along the z (vertical) axis. A second short dipole has its center on the y-axis 2km distance from the first dipole, is also aligned with the z-axis, and a voltage of 1mV is measured by the second dipole. The second dipole is then elevated 1km (placing its center at \( x=0, y=2\text{km}, z=1\text{km} \)), and aligned with the z-axis. What is the new voltage measured by the second dipole? (8 points)

\[
V_0 \text{ Haye drops by a factor of } \frac{\sin \Theta}{n_1/n_0} = \frac{(0.894)^2}{\sqrt{5}} = 0.35 \text{ mV}
\]

\[
V = 0.35 \text{ mV}
\]

2. A uniform plane wave in air, traveling in the z-direction, with normal incidence to a dielectric wall has a total electric field, \( |E^{\text{total}}| \), as shown in the figure below. (8 points)

\[
|E_{\text{total}}| = \left| 1 + \frac{R}{2} \right| E_{\text{total}}
\]

\[
R = \frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon_d}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon_d}} = \frac{1}{2}
\]

\[
-2(\sqrt{\varepsilon_0} - \sqrt{\varepsilon_d}) = \sqrt{\varepsilon_0} + \sqrt{\varepsilon_d}
\]

\[
T \varepsilon_d = 3 \varepsilon_0
\]

What is the permittivity, \( \varepsilon \), of the dielectric wall?
3. Two isotropic oscillators are placed on the y-axis, at $y = -\lambda/4$ and $y = \lambda/4$, with an intrinsic phase shift of $\psi = \pi$. Given that the azimuthal angle, $\phi$, is defined as the angle from the x-axis, determine all values of $\phi$ which give nulls (zeroes) for the electric field pattern in the x-y plane. (8 points)

$$d = \frac{\lambda}{2}, \quad kL = \frac{\pi}{2}, \quad \phi = \frac{\pi}{2}, \quad \theta = \frac{\pi}{2} \cos \psi + \frac{\pi}{2}, \quad \cos \theta = \sin \phi$$

Find zeroes of:

$$\frac{\sin N \frac{\pi}{2}}{N \sin \frac{\pi}{2}} = \frac{\sin (\pi \sin \phi + \pi)}{2 \sin (\frac{\pi}{2} \sin \phi + \pi)}$$

Zeroes @ $\phi = 0, \pi$

$$\text{nulls} = \frac{0, \pi}{2}$$

4. A dipole of length $2\lambda$ is oriented along the z-axis. Determine all values of $\theta$ which give nulls (zeroes) in the electric field pattern. (8 points)

$$|E| = \frac{\cos \left( \frac{k \theta \cos \theta}{2} \right) - \cos \left( \frac{k \theta}{2} \right)}{\sin \theta} = \frac{\cos \left( 2 \pi \cos \theta \right) - 1}{\sin \theta}$$

Try $\theta = 0, I|E| = 0$ or L'Hopital's rule

$$\frac{\partial}{\partial \theta} \left( \cos \left( 2 \pi \cos \theta \right) \right) = 2 \pi \sin \theta \sin \left( 2 \pi \cos \theta \right) \cos \theta$$

Try $\theta = \pm \frac{\pi}{2}$, $|E| \to 0$

$$|E| \to 0 \text{ as } \theta \to 0 \text{ or } \pi$$

$$\text{nulls} = \frac{0, \pm \frac{\pi}{2}, \pi}{2} \text{ (four nulls)}$$
5. Two half-wavelength dipoles are aligned parallel to the y-axis, but have their centers located at \( x = \pm \lambda/4 \) on the x-axis (so that they are separated by \( \lambda/2 \)). Assume that they are driven in phase, with equal amplitudes. For the \( x-y \) plane, on the axes provided, sketch the element pattern, the array factor and the total (far-field) pattern (using pattern multiplication) (3 points for each sketch). Note that the sketches must show the correct placement of the nulls (zeroes), but need not otherwise be quantitatively perfect.