Assume an isotropic oscillator:

\[ |\vec{A}| = I_o \frac{e^{-jkr}}{4\pi r} \]

Array example #1

Next consider two such oscillators with equal amplitudes, equal phase and frequency separated by \( d = \lambda/2 \):
The array factor is then:

$$AF = 1 + e^{-jkd\cos\gamma}$$

$$= e^{-j\frac{kd\cos\gamma}{2}} \left( e^{j\frac{kd\cos\gamma}{2}} + e^{-j\frac{kd\cos\gamma}{2}} \right)$$

where $kd = 2\pi d/\lambda = \pi$. Normalized:

$$f(\gamma) = \cos \left( \frac{\pi}{2} \cos\gamma \right)$$
\( n = 2, d = \lambda / 2, \Psi = 0 \)
Array example #2

Next consider two such oscillators with equal amplitudes, equal frequency, and with phase shift $\Psi = \pi/2$, separated by $d = \lambda/4$.
The array factor is then:

\[ AF = 1 + e^{-jkd\cos\gamma} e^{j\pi/2} \]
\[ = e^{-j\frac{kd\cos\gamma}{2}} e^{j\pi/4} \left( e^{j\left(\frac{kd\cos\gamma}{2} - \frac{\pi}{4}\right)} + e^{-j\left(\frac{kd\cos\gamma}{2} - \frac{\pi}{4}\right)} \right) \]

where \( kd = 2\pi d/\lambda = \pi/2 \). Normalized this becomes:

\[ f(\gamma) = \cos \left(\frac{\pi}{4}(\cos\gamma - 1)\right) \]
$n = 2, d = \lambda/4, \Psi = \pi/2$
Uniform linear arrays

For $N$ oscillators, equal frequency, equal amplitude, equal spacing, $d$, and a possible progressive phase shift, $\Psi$, the array factor is then:

$$AF = A_0 \sum_{n=0}^{N-1} e^{jn\alpha}$$

$$= A_0 \frac{1 - e^{jN\alpha}}{1 - e^{j\alpha}}$$

$$= A_0 e^{j(N-1)\alpha/2} \frac{\sin(N\alpha/2)}{\sin(\alpha/2)}$$

where $\alpha \equiv kdcos\gamma + \Psi$ and normalized becomes:

$$f(\gamma) = \frac{\sin(N\alpha/2)}{N\sin(\alpha/2)}$$
$n = 2, d = \lambda/2, \Psi = \pi$
\[ n = 2, d = \lambda/2, \Psi = \pi/2 \]
\( n = 2, d = \lambda, \Psi = 0 \)
$n = 10, d = \lambda/2, \Psi = 0$