Show your work for full credit. Useful constants:

\[
\begin{align*}
\epsilon_0 & = 8.854 \times 10^{-12} \text{ F/m} \\
\mu_0 & = 4\pi \times 10^{-7} \text{ H/m}
\end{align*}
\]

1. The amplitude of \( \vec{E} \) just inside a liquid is 1.0V/m and \( \text{Re}\{\epsilon\} = 10 \times \epsilon_o \), with \( \sigma = 0.5 \text{mho/m} \). Determine the amplitude of \( \vec{E} \) at a distance of 0.1m inside the medium for \( f=10\text{GHz} \) (\( 1 \times 10^{10} \) Hz). (5 points)

Solution:

\[
k = \omega \sqrt{\mu \epsilon} \sqrt{1 - j \frac{\sigma}{\omega \epsilon}} = 663.42 - j29.75
\]

Note that \( \frac{\sigma}{\omega \epsilon} = 0.09 \ll 1 \) so \( k_I = \frac{1}{2} \sigma \sqrt{\mu/\epsilon} = 29.78 \). Then, \( 1.0e^{-k_I(0.1)} = 0.051[V/m] \)

2. Given \( \vec{E} = ((1 - j)\hat{x} + (1 + j)\hat{z}) \exp^{-j\nu} \),

(a) What direction is the wave traveling? (2 points)

Ans.: +y -direction

(b) What polarization does this wave have, linear, circular, or elliptical (with handedness, left- or right-)? (3 points)

Solution:

\( \hat{z} \times \hat{x} = \hat{y} \) so \( \frac{E_y}{E_0} = \frac{1-j}{1+j} = 1.0 \angle -90^\circ \)

circular, right-handed (magnitudes of \( \hat{x}, \hat{z} \) equal and \( \phi = -\pi/2 \)