1. Given a rectangular waveguide air-filled with dimensions $a=2.29\text{cm}$, $b=1.02\text{cm}$, what operating frequency would be chosen if we wished to operate at the average of the two lowest propagating (cut-off) frequencies? (5 points)

**Solution:**

$$f_c = \frac{1}{2\sqrt{\mu_0\epsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3 \times 10^8}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Therefore:

$TE_{10} = 6.55\text{GHz}$

$TE_{20} = 13.10\text{GHz}$

$TE_{01} = 14.71\text{GHz}$

So, average of two lowest is: \(\frac{(6.55 + 13.10)}{2} = 9.83\text{GHz}\)

2. What is the lowest frequency that can be propagated in the earth-ionosphere waveguide, assuming the ionosphere is 50 km from the surface of the earth? Your answer must include the mode. (5 points)

**Solution:**

The $TM_0$ (or $TEM$) mode has a cutoff frequency of 0Hz.

For the $TE$ mode,

$$f_c = \frac{m}{2a\sqrt{\mu_0\epsilon_0}}$$

Set $m = 1$. Then:

$$f_c = \frac{1}{2(50 \times 10^3 m)}3 \times 10^8[m/s]$$

$$= \frac{3}{100} \times 10^5$$

$$= 3kHz$$

Therefore, the lowest propagated frequency is 0Hz and is the $TM_0$ (or $TEM$) mode.