EE4411 First Hour Exam, Fall 2015

Each problem is worth 5 points. All units are mks and are considered part of the answer. Show your work for full credit.

1. A time-harmonic electromagnetic field in free space is perpendicularly incident upon a perfectly conducting surface with normal unit vector $\hat{y}$ and incident electric field $\vec{E}_i = \hat{x}E_0 \exp^{-j\beta y} [V/m]$. Find the induced surface current $\vec{J}$ and reflected electric field $\vec{E}_r$.

$$\vec{E}_r = -\hat{x}E_0 e^{\frac{-j\beta y}{\beta}} [V/m]$$

$$\overrightarrow{H}_i = \frac{-1}{j\omega\mu_0} \nabla \times \vec{E} = \frac{j}{\omega\mu_0} (-\frac{\partial}{\partial y} E_0 e^{-j\beta y}) = -\frac{\beta E_0 e^{-j\beta y}}{\omega\mu_0} \hat{z} \left[ \frac{A}{m} \right]$$

$$\vec{J}_s = \hat{n} \times (\overrightarrow{H}_i + \overrightarrow{H}_r) \bigg|_{y=0} = -\hat{y} \times \hat{z} \left( \frac{-\beta E_0}{\omega\mu_0} \right) \hat{z} = \hat{n} \frac{E_0}{\beta} \frac{2}{\pi} \left[ \frac{A}{m} \right]$$

2. Give a general solution for the electric field for the wave equation which is a standing wave in the x-direction, a standing wave in the y-direction, and is propagating in the z-direction.

$$E_x = (A \cos \beta_x x + B \sin \beta_x x)(C \cos \beta_y y + D \sin \beta_y y) \exp^{-j\beta_z z}$$
3. Given the magnetic field \( \vec{H} = (x + j y) \cos(\beta, z) \), find the time-average power density of the wave, \( \vec{S}_{av} \), flowing in the z-direction.

\[
\vec{S}_{av} = \frac{1}{2} \Re \left\{ \vec{E} \times \vec{H}^* \right\} , \quad \vec{E} = \frac{\nabla \times \vec{H}}{j \omega \varepsilon} = \frac{1}{j \omega \varepsilon} \begin{bmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
\frac{x}{2} & \frac{y}{2} & \frac{z}{2} \\
\cos \beta z & \sin \beta z & \theta
\end{bmatrix}
\]

\[
= \frac{1}{j \omega \varepsilon} \begin{bmatrix}
-\frac{x}{2} j \beta \sin \beta z + \frac{y}{2} \beta \cos \beta z
\end{bmatrix}
\]

so \( \vec{S}_{av} = \frac{1}{2} \Re \left\{ \vec{E} \times \vec{H}^* \right\} = \frac{1}{2 \omega \varepsilon} \frac{\beta \sin \beta z \cos \beta z}{\Re \left\{ \frac{\beta}{2} \right\} \left\{ \frac{\beta}{2} \right\} } = 0
\]

4. Consider a conducting plane in the \( x' - z' \) plane, with a rectangular aperture with dimensions \(-a/2 \leq x' \leq a/2\) and \(-b/2 \leq z' \leq b/2\). Given that the electric field in the aperture is:

\( \vec{E}_a = \hat{x} E_o \),

(a) Find \( r' \cos \psi \), where \( \psi \) is the angle between the ray to the source, \( \vec{r}' \), and the ray to the observer, \( \hat{r} \) (as always).

\[
r' \cos \psi = \left( \hat{x} \times \hat{z} \right) \cdot (x' \sin \theta \cos \phi + y' \sin \theta \sin \phi + z' \cos \theta)
\]

\[
= x' \sin \theta \cos \phi + y' \cos \theta
\]

(b) Find \( ds' \), the differential surface element corresponding to the source current.

\[
ds' = dx' \, dz'
\]