EE4411 Hour Exam II, Fall 2015

Each problem is worth 5 points. All units are mks and are considered part of the answer. Show your work for full credit.

1. A small circular loop of radius $a$ lying in a $y-z$ plane with its center at $x=0$, $y=0$ and $z=h$, where $h > a$. The loop carries a current which flows clockwise as viewed along the $+x$-axis. If the $z=0$ plane is a perfect conductor, sketch the loop with its current and also the current flow of the image.

2. A parallel-plate waveguide is formed by placing two infinite planar conductors at $y=0$ and $y=b$. For a TE wave, the electric field is given by: $E_x = E_0 \sin(\beta_y y) e^{-\gamma z}$. Find the allowed values of $\beta_y$ and the cutoff frequencies.

$$F_y = 0 \quad \text{at} \quad y = b, \quad \beta_y b = m\pi$$

or $\beta_y = \frac{m\pi}{b} \quad ; \quad m = 1, 2, 3 \ldots$

To find cutoff, $\gamma = \sqrt{\beta_y^2 - \beta_c^2} = 0$

or $\beta_c = \beta = \omega \mu \varepsilon_0 = \frac{\omega}{c} = \frac{m\pi}{b}$

$$\omega_c = \frac{m\pi}{b c} \quad \therefore f_c = \frac{m}{2bc}$$

($c =$ speed of light)
3. Design a circular waveguide filled with a lossless dielectric medium with \( \varepsilon_r = 2.0 \). The waveguide must operate in a single mode over a bandwidth of 1GHz. Find the radius, \( a \).

\[
\Delta f = (f_c)_{01}^{TM} - (f_c)_{11}^{TE} = \frac{\chi_{01} - \chi_{n1}'}{2\pi a} \frac{c}{\sqrt{\varepsilon_r \varepsilon_0}} = 1 \times 10^9 \text{ Hz}
\]

\[
\therefore a = \frac{(\chi_{01} - \chi_{n1}')}{2\pi c \sqrt{\varepsilon_r (1 \times 10^9)}} = \frac{(2.744 - 1.8112) \times 10^9}{2\pi \times 0.01 \times (1 \times 10^9)}
\]

\[
= 0.019 \text{ m} = 1.9 \text{ cm}
\]

4. Assume a spherical cavity with 1cm radius, filled with air. Determine the resonant frequency of the dominant degenerate modes and the bandwidth of the dominant degenerate modes before the next higher-order modes.

\[
(f_c)_{01}^{TM} = \frac{2.744}{2\pi a \sqrt{\mu_0 \varepsilon_0}} = \frac{2.744}{2\pi (0.01)} = 13.1 \text{ GHz}
\]

Next mode \( + \):

\[
\frac{(f_c)_{021}^{TM}}{(f_c)_{01}^{TM}} = \frac{3.870}{2.744} = 1.41 \quad \text{(or \ B W = } \Delta f = f_{upper} - f_{lower} = (1.41 - 1) 13.1 \text{ GHz} = 5.37 \text{ GHz} \text{)}
\]